Where Less Is More: Reducing Variable Annuity Fees to Benefit Policyholder and Insurer

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Abstract

After two decades of increasing popularity, sales of variable annuities (VAs) began to dwindle in 2013. Financial advisors have long argued against investing in VAs due to the product's high fees. VA providers charge these fees—typically at a constant rate throughout the policy period—to cover their expenses and the costs of the embedded guarantees, and lowering this constant fee rate could make the VA unprofitable. Instead, we propose and analyze a simple change to the fee *structure* that would lower overall fees (and thus make the product more attractive to investors) without reducing the insurer's profit. In fact, this time-dependent fee structure—whereby the fee rate is reduced significantly only after a specified number of policy years—can be Pareto-improving for both parties.

The key insight is that the new fee structure discourages policy exchanges (by introducing an implicit cost), which reduces the insurer's policy acquisition expenses.

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Taking into account financially optimal lapse (and reentry) decisions, we determine the optimal timing and rate of the fee reduction for a competitive as well as for an innovative VA provider. An important characteristic of this feature is that it can be implemented easily and effectively to both new and existing VA policies.

Keywords: Variable annuities, pricing, GMDB, fees, commissions, surrender behavior.

1 Introduction

Over the past two decades, Variable Annuities (VAs) have developed into highly popular long-term investment vehicles. In the U.S., these products combine the investment features of mutual funds with favorable tax treatment and return guarantees (Hardy, 2003). Recently, however, VA sales have started to dwindle. Since 2013, VA providers have experienced negative net sales,[1] at an increasing rate. In fact, financial advisers have long tried to steer consumers away from VAs, largely due to the high fees associated with the product (see e.g. NASDAQ (2009); Kiplinger (2011); The Wall Street Journal (2012); Forbes (2015), among many others).[2]

To counteract the recent decline in demand, VA providers need to find ways to make the product more attractive to investors. Reducing their fees would appear to be an effective way to accomplishing that. However, the reason for the high fees—which are typically charged at a constant rate throughout the policy period—is a combination of large policy acquisition expenses (e.g. commissions) and frequent policy lapses, which give providers less time to recover their up-front expenses (Pinquet, Guillen, and Ayuso, 2011; Moenig and Zhu, 2016). Therefore, simply reducing the constant fee rate is likely not a viable option. Instead, we suggest that the solution to the provider's demand problem could be a change in the fee *structure*, whereby fees decline over time in order to reward and encourage long-term participation in the policy.

[1] That is, investors are surrendering policies faster than new money is coming into the market. Source: Insured Retirement Institute.

[2] Research shows that the tax benefits of VAs can outweigh these fees, but only in the long term (Milevskya and Panyagometha, 2001; Moenig and Bauer, 2015; Moenig and Zhu, 2016).

We implement a VA with a basic death benefit guarantee and a binary fee structure based on a one-time fee reduction. We find that this simple time-dependent fee structure suffices to effectively discourage policy lapses. This in turn lowers the insurer's (per-year) expenses and therefore the policyholder's average annual fee—up to 35% (that is 8% of the total premium) under our model specifications. Such a policy would be substantially more attractive for potential investors. Moreover, we find that there is also a lot to gain for an innovative VA provider willing to deviate from the current status-quo—a surplus of around 6% of the total premium, according to our model. That is, our proposed fee structure can make both policyholder and insurer better off.

The policyholder's incentive to lapse stems from the fact that the death benefit guarantee commonly embedded in VAs (or any other typical VA guarantee, for that matter) resembles a (conditional, long-term) put option on the VA account value, and therefore loses in value if the account value increases. Since the policyholder continues to pay fees for this now overvalued guarantee, he would benefit from lapsing the current VA policy and immediately "re-entering" the market by purchasing the identical product. This would increase the guaranteed amount to the current VA account value, without changing any of the other contract parameters. Moenig and Zhu (2016) show that this "lapse-and-reentry" strategy—known commonly as a *1035-exchange*—is frequently optimal and persists even in the presence of a typical VA surrender fee schedule. The strategy is quite costly, though, since the policyholder's market reentry constitutes the sale of a new policy. The resulting policy acquisition expenses are borne initially by the insurer—and (in anticipation) apportioned to the policyholder in the form of a larger VA fee. As such, the costs of the policy lapse do not have a direct effect on the policyholder's decision making. In fact, the frequency of lapses is a consequence of the fact that in the current VA market setting there is often no cost to lapsing. Our proposed time-dependent fee structure changes that, as lapsing the VA policy would result in (temporarily) foregoing the reduced fee rate.

A key feature of our proposed fee structure is that it is very easy to implement by VA providers. It can be included in new VA products, but can also be assessed retroactively on existing policies. In addition, policyholders should easily be able to understand the new feature and realize that it is beneficial to them, and ultimately factor it into their lapse decision making. Given that our reduced fee leads to an improved outcome for both the insurer and the policyholder, our proposal provides thus an immediate competitive advantage to any insurer willing to adopt it. We thus believe that the proposed fee structure could have a significant positive impact on the U.S. VA industry. In addition, the resulting reduction in lapses is beneficial for various reasons not captured in our model: For instance, policy surrenders complicate the hedging of the insurer's financial risk exposure (Kling, Ruez, and Ruß, 2014) and force the insurer to remain liquid at all times (Gollier, 2015; Kuo, Tsai, and Chen, 2003).

Extensive research has been conducted on pricing VA guarantees and on modeling optimal exercise by policyholders (see e.g., Milevsky and Posner (2001), Bauer, Kling, and Russ (2008), Dai, Kuen Kwok, and Zong (2008), Bernard, MacKay, and Muehlbeyer (2014), among many others.) The consensus is that policyholder behavior poses a tremendous risk for VA providers. Our work builds on recent studies that analyze some specific VA policy features regarding their effectiveness in mitigating incentives to surrender (MacKay, Augustyniak, Bernard, and Hardy, 2015; Moenig and Zhu, 2016). One effective design to reduce incentives to surrender is to include embedded ratchet options within the VA policy. The idea is then to update the guaranteed level periodically throughout the life of the policy so that the VA policy stays attractive even when the underlying fund value is large and the maturity guaranteed level is too low in comparison (too deep out of the money guarantee). But ratchet options are highly path-dependent and known to be difficult to hedge. Another effective design is to adjust the constant fee to be paid over the years of the contract so that it depends on market conditions. Such a *state-dependent* fee structure was proposed in Bernard, Hardy, and MacKay (2013), where the fee is paid to the insurer only when the account value is below a certain threshold (see also Delong (2014); Zhou and Wu (2015); Moenig and Zhu (2016)). It is very effective in reducing lapse incentives. However, this type of fee is controversial as it may give insurers incentives to maintain the fund below the threshold to keep receiving fees, and thus does not give incentives to maximize the fund value as they do not get any income when the fund is above the threshold. Furthermore, this type of fee may also be subject to manipulation when the fund value is close to the threshold due to the discontinuity in the fee structure that is directly related to the performance of the fund, which is managed by the VA provider. We see an advantage in the time-dependent fee structure as it benefits all policyholders, not only those who already enjoy a positive investment performance. In addition, the discontinuity in the payment of the fee is driven by time and not subject to any manipulation, which makes it also a very easy design to understand.

The remainder of this article is organized as follows. We begin by presenting a financial model of a VA with a simple time-dependent fee structure. The two sections that follow assess the benefits of this fee structure, first for an innovative insurer, then for a competitive VA market where all insurers have adopted this fee structure. In particular, we discuss the optimal amount by which the fee should be reduced and the optimal time at which to reduce the fee. Lastly, we offer conclusions and ideas for future research.

2 VA Pricing Model with a Time-Dependent Fee Structure

We extend the lapse-and-reentry model of Moenig and Zhu (2016) by allowing the annual VA fee to be time-dependent. We first recall here the assumptions made regarding the financial market and present the VA contract that we will study throughout the paper. We then provide details on the valuation procedures, from the perspective of both policyholder and insurer. Lastly, we discuss our numerical implementation.

2.1 Model Setup

A typical VA policy consists of two phases. First, the policyholder makes one or more payments into a fund managed by the insurer until the maturity date T (accumulation phase). Then, he receives income from the insurance company based on his accumulated account value and possibly subject to some minimum guarantees (payout phase). Specifically, we consider a single-premium VA policy whereby the policyholder invests an amount A_0 with an insurance company at time 0 but makes no further payments. The investment is placed in an equity fund whose evolution follows a Geometric Brownian motion:

$$dS_t \;=\; \mu\, S_t\, dt + \sigma\, S_t\, dZ_t\;,\;\; S_0 > 0\;, \tag{1}$$

with drift parameter μ and volatility σ, and where $(Z_t)_{t>0}$ follows a standard Brownian motion.[3] We denote the account value of the VA investment at time t by A_t. At maturity (time T), the insurer pays out the accumulated account value A_T.[4]

As is common practice in the U.S., we include in the VA policy a return-of-premium GMDB rider, a surrender fee schedule, and the right to lapse the policy during the accumulation phase. Under the GMDB, the insurer guarantees to pay out the larger of the VA account value A_t and a guaranteed amount G_t—beginning with the initial investment: $G_1 = A_0$—upon the policyholder's death. That is, if death occurs in the t-th policy year (for $t \in \{1, \dots, T\}$), the policyholder receives $\max\{A_t, G_t\}$ at time t, with A_t coming from his VA account, while the remainder, $\max\{G_t - A_t, 0\}$, is supplemented by the insurer.

The policyholder may lapse his VA contract on policy anniversary dates, that is at times $t = 1, 2, \dots, T-1$. Upon lapsing at time t, he receives the current VA account value net of surrender fees, that is

$$A_t^{\text{lapse}} \;:=\; [1 - s(m_t)]\, A_t\;,$$

where m_t denotes the time (in years) since inception of the current VA policy (that is, since his most recent policy lapse), and $s(m_t)$ is the applicable surrender fee after m_t contract years. For instance, if the policyholder lapses for the first time at time t, we have $m_t = t$, and $m_{t+1} = 1$. Further, we let n_s denote the length of the surrender schedule, that is the

[3]This setting is the well-known Black-Scholes model. In our numerical implementation we take advantage of its simplicity. We are aware of its limitations in reflecting empirical stock returns—however, we believe that the impact of applying more sophisticated financial models (accounting, e.g., for stochastic volatility and stochastic interest rates) would have merely a second-order effect relative to the policyholder behavior we study in this article (see Kling, Ruez, and Ruß (2014)).

[4]Whether this occurs as a lump sum payout or in the form of an annuity stream is immaterial to our model since both have the same present value A_T at time T.

number of initial contract years in which there is a positive surrender fee:

$$n_s \;=\; \min_{n\in\mathbb{N}} \{s(n)=0\} \; .$$

Following the lapse, the policyholder immediately reinvests the amount A_t^{lapse} into an identical product (same insurer, same year of maturity); as a result, the guaranteed amount for this new policy will also be equal to A_t^{lapse}, and the surrender fee schedule starts over:

$$G_{t+1} \;=\; A_t^{\text{lapse}} \qquad \text{and} \qquad m_{t+1} \;=\; 1 \, .$$

The insurer incurs two types of expenses: a policy acquisition expense (at rate ε_{ini}) that is assessed at the beginning of the policy—including the policyholder's market reentry following a lapse—and that accounts for commissions, marketing, and administrative costs; and recurring expenses (at rate ε_{rec}), which are assessed at the beginning of each year during the accumulation phase. Both expense rates are determined in proportion to the VA account value at the time.

To cover its costs for expenses and the GMDB rider, the insurer charges a recurring fee at annual rate ϕ_{m_t}, assessed continuously and in proportion to the current VA account value A_t at time t. The fee is taken directly out of the VA account. In the U.S. VA market, ϕ_{m_t} is typically independent of m_t. By contrast, we propose the following time-dependent fee:

$$\phi_{m_t} \;=\; \begin{cases} \phi_{\text{ini}} \, , & m_t < n_{\text{red}} \\ \phi_{\text{red}} \, , & m_t \geqslant n_{\text{red}} \end{cases} \tag{2}$$

where ϕ_{ini}, ϕ_{red}, and n_{red} are all specified in the VA contract. n_{red} denotes the number of policy years after which the fee is reduced. For simplicity, let $\hat{\phi}$ denote the three-dimensional vector containing the policy's fee information:

$$\hat{\phi} = [\phi_{\text{ini}}, \phi_{\text{red}}, n_{\text{red}}]. \tag{3}$$

Lastly, and in line with the actuarial literature in this context, we assume independence between financial market risk and individual mortality risk. Specifically, we define the probability measure $\mathbb{Q}$ as the product measure of the risk-neutral measure for financial risk and the real-world measure for (idiosyncratic) mortality risk, while $\mathbb{P}$ is the product measure of the real-world measures for both financial and mortality risk. Regarding the latter, we let q_x denote the probability that a person age x dies within the following year, while $p_x = 1 - q_x$ represents the probability that an x-year old policyholder survives another year.

Our work builds on the lapse-and-reentry model developed by Moenig and Zhu (2016). In this paper, the authors show that the policyholder's optimal behavior is largely driven by value maximization, and unaffected by tax considerations and market incompleteness. This value-maximization approach is in line with most of the existing literature on policyholder behavior in VAs (see e.g. Milevsky and Salisbury (2001); Bauer, Kling, and Russ (2008); Bernard, MacKay, and Muehlbeyer (2014)); however, it is important to account for the relevant market frictions (in this case, the policy acquisition expenses) in order to reconcile the model with typical market prices. Moreover, Moenig and Zhu (2016) are able to justify consumer participation once they include proper tax treatments and individual risk preferences in their model. These results naturally extend to our approach and justify our use of value maximization as an appropriate methodology for the policyholder.

2.2 Optimal Lapse-and-Reentry

We assume that the policyholder maximizes the value of his VA policy in that his lapse decision at each policy anniversary date serves to maximize the market value of his investment (expressed as a risk-neutral expected value under $\mathbb{Q}$). This market value depends on the current account value A_t, the guaranteed amount G_t, and the time m_t since inception of the current VA policy (m_t is an integer value that denotes the number of full years into the policy). Note that the latter impacts both the potential time-t surrender fee $s(m_t)$ and the VA fee rate ϕ_{m_t} for the coming year. For integer values of t, and assuming that the policyholder is still alive at time t, we denote by $V_t(A_t, G_t, m_t)$ the market value of the VA policy immediately prior to the policyholder's time-t lapse decision.

For notational convenience we first define the intermediary function

$$
\begin{aligned}
\widetilde{V}_t(A_t, G_{t+1}, m_t) \;&=\; q_{x+t}\left[A_t\, e^{-\phi_{m_t}} + \mathrm{Put}(A_t, G_{t+1}, \phi_{m_t})\right] \\
&\quad + (1-q_{x+t})\, e^{-r}\mathbb{E}_t^{\mathbb{Q}}\left[V_{t+1}(A_{t+1}, G_{t+1}, 1+m_t)\right]\;,
\end{aligned}
$$

where the VA fee rate ϕ_{m_t} is determined by Equation (2), $\mathbb{E}_t^{\mathbb{Q}}[.]$ denotes the expected value—under the measure $\mathbb{Q}$—based on the information available at time t, the VA account value is updated according to

$$
\begin{aligned}
A_{t+1} \;&=\; A_t \exp\left[r - \phi_{m_t} - \tfrac{1}{2}\,\sigma^2 + \sigma\,\left(Z_{t+1} - Z_t\right)\right]\;, \\
Z_{t+1} - Z_t \;&\sim\; \mathcal{N}(0,1)\,,
\end{aligned}
\tag{4}
$$

and where we denote by $\mathrm{Put}(S_0, K, \phi)$ the Black-Scholes price of a one-year put option with current stock price S_0, strike price K, and dividend yield ϕ:

$$
\begin{aligned}
\mathrm{Put}(S_0, K, \phi) \;&=\; K\, e^{-r}\,\mathcal{N}(-d_2) - S_0\, e^{-\phi}\,\mathcal{N}(-d_1)\;, \text{with} \\
d_1 \;&=\; \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \phi + \frac{\sigma^2}{2}\right)}{\sigma} \quad \text{and} \quad d_2 \;=\; d_1 - \sigma.
\end{aligned}
\tag{5}
$$

In particular, we take advantage of the fact that the dividend yield has the same impact on the stock price as the continuously deducted fee rate ϕ_{m_t} has on the VA account value A_t.

If the policyholder lapses at time t, the state variables are set to $A_t = A_t^{\text{lapse}}$, $G_t = A_t^{\text{lapse}}$, and $m_t = 0$. The policyholder will lapse if and only if the VA value upon lapse-and-reentry, defined by

$$
V_t^{lapse}(A_t, G_t, m_t) \;=\; \widetilde{V}_t\left(A_t^{\text{lapse}}, A_t^{\text{lapse}}, 0\right)\;,
$$

exceeds the continuation value of the policy, defined by

$$
V_t^{cont}(A_t, G_t, m_t) \;=\; \widetilde{V}_t(A_t, G_t, m_t)\;.
$$

. The implicit value of the VA policy is then given by the larger of the two values:

$$V_t(A_t, G_t, m_t) \;=\; \max\left\{V_t^{cont}(A_t, G_t, m_t), V_t^{lapse}(A_t, G_t, m_t)\right\} \;.$$

The resulting dynamic optimization problem can be solved recursively using the terminal condition

$$V_T(A_T, G_T, m_T) \;=\; [1 - s(m_T)]A_T \;. \tag{6}$$

Ultimately (by slight abuse of notation) as

$$V_0 \;:=\; \widetilde{V}_0(A_0, A_0, 0) \;. \tag{7}$$

V_0 constitutes the expected present value of the VA policy to the policyholder.

2.3 Insurer's Valuation

By contrast, the initial *net present value* of a VA policy to the insurer is given by:

$$NPV_0 \;=\; NPV_0(\hat{\phi}) \;=\; A_0 - V_0 - EPVE_0 \;, \tag{8}$$

where V_0 is given by (7) and $EPVE_0$ denote the time-0 expected present values of expenses. Note that both V_0 and $EPVE_0$ also depend on $\hat{\phi}$ (defined in (3)) but we omit this dependence for the ease of presentation.

To compute $EPVE_0$, we proceed recursively as follows. For integer values of t, we let $EPVE_t(A_t, G_t, m_t)$ denote the time-t expected present values of all expense payouts from time t forward. To compute $EPVE_t(A_t, G_t, m_t)$, we then define the following intermediary function:

$$\widetilde{EPVE}_t(A_t, G_{t+1}, m_t, \varepsilon) \;=\; \varepsilon A_t + (1 - q_{x+t})\, e^{-r}\mathbb{E}_t^{\mathbb{Q}}\left[EPVE_{t+1}\left(A_{t+1}, G_{t+1}, 1 + m_t\right)\right] \;.$$

Here, ε refers to the applicable expense rate for the upcoming year. The recursion procedure has terminal condition

$$EPVE_T(A_T, G_T, m_T) \;=\; 0\,.$$

From there, we proceed recursively for times $t = T-1, T-2, \ldots, 1$ and for given A_t, G_t, m_t to define

$$EPVE_t(A_t, G_t, m_t) = \begin{cases} \widetilde{EPVE}_t\left(A_t^{\text{lapse}}, A_t^{\text{lapse}}, 0, \varepsilon_{\text{ini}} + \varepsilon_{\text{rec}}\right)\,, \\ \qquad\qquad\qquad \text{if } V_t^{lapse}(A_t, G_t, m_t) > V_t^{cont}(A_t, G_t, m_t) \\ \widetilde{EPVE}_t\left(A_t, G_t, m_t, \varepsilon_{\text{rec}}\right)\,, \\ \qquad\qquad\qquad \text{if } V_t^{lapse}(A_t, G_t, m_t) \leqslant V_t^{cont}(A_t, G_t, m_t) \end{cases} \tag{9}$$

Ultimately, we find the desired quantity $EPVE_0$, as a function of the VA fee structure:

$$EPVE_0 \;=\; \widetilde{EPVE}_0\left(A_0, A_0, 0, \varepsilon_{\text{ini}} + \varepsilon_{\text{rec}}\right)\,.$$

2.4 Expected Number of Policy Lapses

We can determine the expected number of lapses (under the real-world measure $\mathbb{P}$) in similar fashion. Thereby, $L_t(A_t, G_t, m_t)$ denotes the expected number of dates over the period $[t, T)$ at which it is optimal for the policyholder to lapse, given the time-t state of the VA policy. The terminal condition is

$$L_T(A_T, G_T, m_T) \;=\; 0\,.$$

For future reference we first define the intermediary function

$$\widetilde{L}_t(A_t, G_{t+1}, m_t) \;=\; (1 - q_{x+t})\,\mathbb{E}_t^{\mathbb{P}}\left[L_{t+1}(A_{t+1}, G_{t+1}, 1 + m_t)\right]\,,$$

subject to the dynamics for the VA account value under the measure $\mathbb{P}$ (that is, Equation (4), but replacing r with μ). We can then find the expected number of lapses recursively for all times and state variables as

$$L_t(A_t, G_t, m_t) = \begin{cases} 1 + \widetilde{L}_t\left(A_t^{\text{lapse}}, A_t^{\text{lapse}}, 0\right) \ , \\ \qquad\qquad \text{if } V_t^{lapse}(A_t, G_t, m_t) > V_t^{cont}(A_t, G_t, m_t) \\ \widetilde{L}_t\left(A_t, G_t, m_t\right) \ , \\ \qquad\qquad \text{if } V_t^{lapse}(A_t, G_t, m_t) \leqslant V_t^{cont}(A_t, G_t, m_t) \end{cases} \tag{10}$$

Proceeding recursively we find the time-0 expected number of policy lapses L_0 as

$$L_0 \ = \ \widetilde{L}_0\left(A_0, A_0, 0\right) \ .$$

2.5 Numerical Analysis

We use recursive dynamic programming to implement the policyholder's optimal control problem along with the insurer's policy valuation and lapse calculations described above. Our state space consists of variables A_t, G_t, and m_t, for time $t = T, T-1, \ldots, 0$. For details, we refer to Moenig and Zhu (2016).

For our numerical analysis of the VA policy and time-dependent fee structure we follow the base-case parameter specifications of Moenig and Zhu (2016), which are summarized in Table 1. That is, we consider an investor who is age 55 and who invests \$100,000 into a VA policy that is set to mature at age 80. The investor's mortality follows the 2012 IAM basic male mortality table. The policy includes a 7-year surrender schedule. We assume a 3% risk-free rate of return, an 8% expected return and 20% annual volatility for the underlying asset, and expenses of 7% of the face amount for policy acquisitions and 0.4% for annually recurring expenses. These expenses are typical for the U.S. VA industry.

Table 1: Parameter Values.

Description	Parameter	Values
Initial VA investment ($)	A_0	100,000
Age at inception (years)	x	55
Time to maturity (years)	T	25
Surrender fees	$s(m_t)$	7%, 6%, ..., 1%, 0, 0, ...
Risk-free rate	r	3%
Expected growth rate of investment	μ	8%
Volatility of investment	σ	20%
Policy acquisition expense	ε_{ini}	7%
Recurring expense rate	ε_{rec}	0.4%

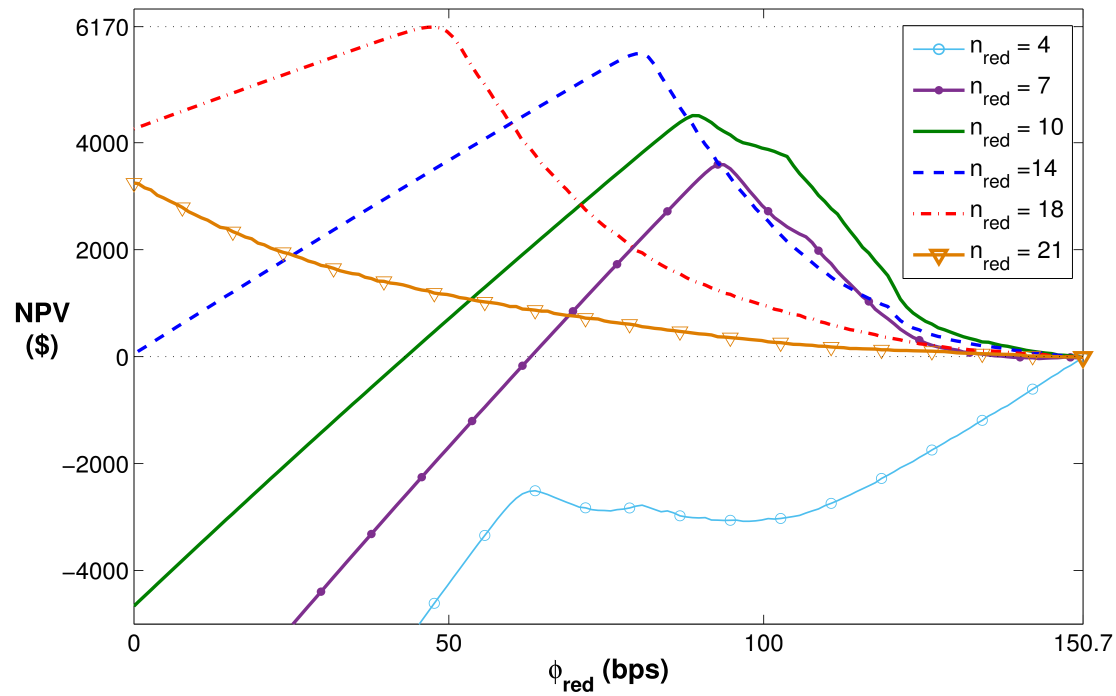
3 Benefits to an Innovative VA Provider

We first consider the current status-quo of the U.S.VA market—that is, a time-invariant fee rate—and explore the financial benefits to an insurance company who (solely) innovates by reducing the fee from the initial rate ϕ_{ini} to the lower rate ϕ_{red}, starting after n_{red} contract years (as described by (2)).

Consistent with Moenig and Zhu (2016) we find that under the current market conditions—and accounting for optimal lapse-and-reentry behavior by the policyholder—the insurer breaks even at a (constant) fee rate of 150.7 bps. We therefore assume in this section that the insurer charges an initial fee rate $\phi_{\text{ini}} = 150.7$ bps, in line with its competitors. However, the innovative company decides to reduce the fee rate to ϕ_{red} after n_{red} contract years, so as to maximize its discounted expected profit from this policy. That is, in this section we fix ϕ_{ini} (at 150.7 bps) and explore combinations of ϕ_{red} and n_{red} in order to maximize NPV_0, as specified in Equation (8).

Clearly, ceteris paribus, the policyholder would welcome such a fee reduction. In addition, we find that the insurer can benefit quite substantially from this strategy as well. Figure 1 displays the insurer's profit as a function of ϕ_{red} and for selected values of n_{red}, while Table 2 provides the corresponding insights numerically.

Figure 1: Time-0 net present value (NPV_0) to the innovative insurer, as a function of ϕ_{red} and for different dates n_{red} of implementation of the reduced fee rate.



Note: The insurer's net present value is calculated in expected present value terms under the measure $\mathbb{Q}$ (see Equation (8)) and is based on $\phi_{\text{ini}} = 150.7$ bps as well as on the parameters displayed in Table 1.

Both Table 2 and Figure 1 show that when reducing the fee rate to $\phi_{\text{red}} = 47.4$ bps after 18 years, the insurer can attain a maximum expected profit of \$6,170 over the 25-year policy period, that is over 6% of the initial investment. At the same time, the policy value increases from \$77,340 to \$78,980. And even if the insurer wanted to reduce the fee sooner (e.g. to 89.4 bps after 10 policy years) in order to make the new product potentially more enticing for a policyholder (V_0 increases to \$80,450), the company would still make a substantial profit (\$4,520 in this case). As a result, both insurer and policyholder would be significantly better off under this time-dependent fee structure.

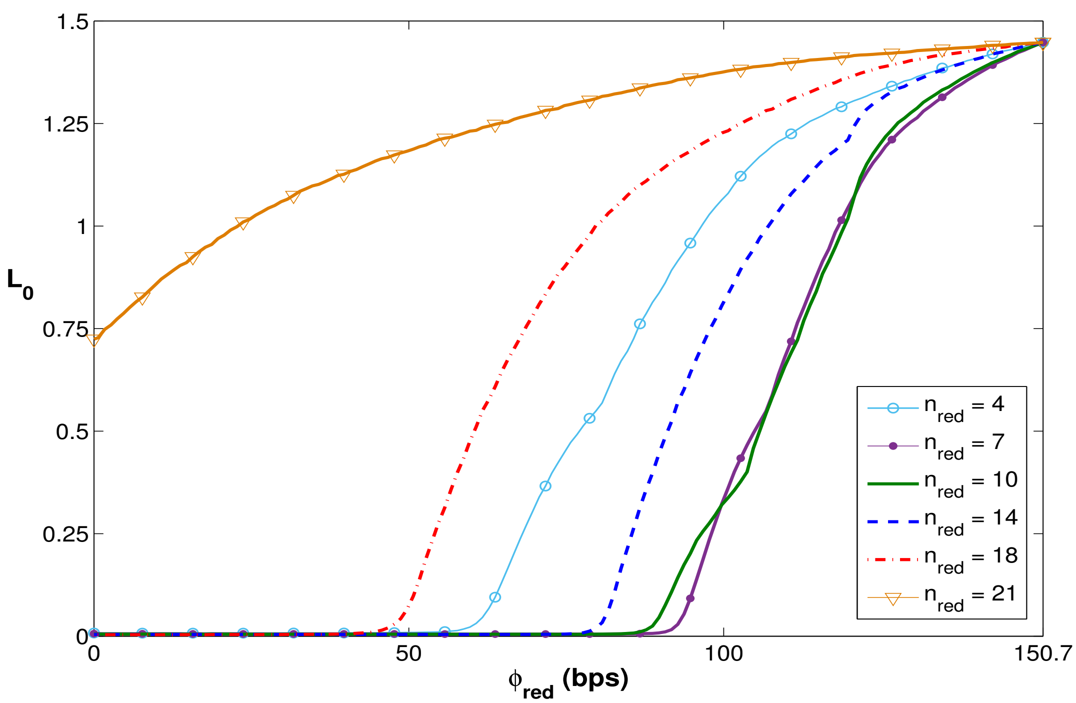
Table 2: Valuation and Lapse Statistics with Innovative Insurer.

	no red.	n_{red}					
		4	7	10	14	18	21
ϕ^*_{red} (bps)	150.7	150.7	93.2	89.4	80.3	47.4	0.1
NPV^*_0 (\$)	0	0	3,600	4,520	5,680	6,170	3,250
V_0 (\$)	77,340	77,340	81,290	80,450	79,420	78,980	78,330
$EPVE_0$ (\$)	22,660	22,660	15,110	15,030	14,500	14,850	18,420
L_0	1.45	1.45	0.04	0.04	0.02	0.02	0.72

Note: For select values of n_{red}, the table depicts the optimal reduced fee rate ϕ^*_{red} and corresponding maximum profit to the innovative insurer (NPV^*_0). It also shows the corresponding policy value V_0, expected expense payment $EPVE_0$, and average number of lapses L_0. Results are based on the parameter values from Table 1 and an initial fee rate $\phi_{\text{ini}} = 150.7$ bps. Lapses are assessed under the measure $\mathbb{P}$, all other values are computed under the measure $\mathbb{Q}$.

The reason for this mutual improvement can be seen in the last two rows of Table 2. We observe that $EPVE_0$ is significantly reduced in the new policy design, and that the average number of lapses L_0 is reduced as well. Figure 2 illustrates further the effect of ϕ_{red} on the number of lapses. The opportunity to pay a reduced fee rate after a certain number of contract years makes the current policy more attractive relative to starting a new policy. This observation shows that there is a hidden cost on policy lapses for policyholders and insurers. Under the current VA market structure the policyholder seems to not really face the most significant cost associated with his policy lapse as the new policy acquisition expenses upon his market reentry (7% of the investment amount at the time) are borne entirely by the VA provider. They are only recovered later in time through the fixed fee percentage taken from the account value—provided that the policyholder maintains the policy for a long period of time. Therefore, the insurer benefits immediately from reduced expenses when the policyholder chooses to lapse less frequently. The fee reduction makes the policyholder face some immediate cost of lapsing as the fee would be increased if he were to lapse and re-enter. In fact, according to Table 2, even if the fee reduction were to take effect only very late in the contract period (e.g. after 18 years), the prospect of a lower fee rate makes lapsing financially optimal only 0.02 times on average over the 25-year period—compared to 1.45 lapses without the fee reduction. This reduces the overall expenses by more than a third (\$14,850 for $n_{\text{red}} = 18$ versus \$22,660 without the fee reduction, see row $EPVE_0$ in Table 2).

Figure 2: Average number of policy lapses (L_0) under the innovative insurer.



Note: Lapse rates are calculated under the measure $\mathbb{P}$ and are based on $\phi_{\text{ini}} = 150.7$ bps as well as on the parameter values displayed in Table 1.

We can see from Figure 2 that reducing ϕ_{red} (below ϕ_{ini} that is equal to 150.7 bps) leads to a substantial reduction in the lapse rate, and thus a substantial reduction in the insurer's expense payments. Reducing ϕ_{red} from ϕ_{ini} to ϕ^*_{red} drives up the insurer's expected profit (see Figure 1).

As we continue to reduce ϕ_{red} below ϕ^*_{red}, the insurer's profit (NPV_0) keeps rising. However, at a certain point the lapse rate reaches approximately zero and further reducing ϕ_{red} will not have an impact on the policyholder's lapse behavior and thus on the insurer's expenses, but only cause the policyholder's fee payments (and thus the insurer's profit) to decline. This is the reason why we find that a moderate reduction in the fee rate is optimal from the insurer's perspective.

The insurer's implicit trade-off between reducing its acquisition expenses (by lowering lapse incentives) and maintaining its fee income is also reflected in its optimal choice of n_{red}. On the one hand, delaying the start of the fee reduction increases the policyholder's overall fee payments; on the other hand it may make him more likely to lapse and reenter. Since the surrender fee sufficiently disincentivizes lapsing during the first seven contract years,[5] there are no reasons for the insurer to choose $n_{\text{red}} < 7$. In those cases, we indeed observe no improvement in lapse rates but only a reduction in profit (see the example of $n_{\text{red}} = 4$ in Table 2 and Figure 2). To the contrary: the insurer benefits from delaying the fee reduction for several years after the surrender schedule ends, as this will allow him to collect more fees without causing a meaningful increase in lapses (note, for instance, that for $\phi_{\text{red}} \approx 45$ bps, lapsing is never optimal for most values of n_{red}). The policyholder, on the other hand, would of course prefer—ceteris paribus—that the fee is reduced as early as possible.

These results demonstrate that any VA provider stands to benefit tremendously from offering this simple, time-dependent fee structure—not only because that would increase the company's per-policy profit, but also because it would make the product more attractive to investors and thus lead to an increased demand. Thereby, a major advantage of this simple adjustment to the VA fee structure is that it can also be applied to existing policies, where it may have an *immediate* positive impact on the financial bottom line of the

[5]See Figure 3 of Moenig and Zhu (2016): under the constant-fee structure, lapsing is never optimal during the first 7 contract years, where the policyholder faces a positive surrender fee. However, at time $t = 7$—when the surrender fee does not apply any longer—the policyholder should lapse in more than half of all financial scenarios.

VA provider: the (prospect of a) reduced fee rate lowers policyholders' incentives to lapse their VA policy and saves the insurer from paying new policy acquisition expenses. We also find that this applies not only to the standard death benefit rider, but also to living benefit guarantees, as we demonstrate in Appendix A for the case of an added Guaranteed Minimum Accumulation Benefit (GMAB) rider.

While our proposed fee structure may offer substantial benefits to an innovative VA provider, the company may not be able to take advantage of them for long. Once other insurers see the value in this new fee structure, they will likely follow suit. Therefore, we explore in the following section what this time-dependent fee structure will lead to in a competitive market environment.

4 Optimal Time-Dependent Fee Structure in a Competitive Market

Initially, an innovative VA provider may want to modify the current market fee structure merely by offering a fee *reduction* (from ϕ_{ini} to ϕ^*_{red}, beginning after a specified number n_{red} of years under contract), but leave the initial fee rate ϕ_{ini} unchanged at the current market level. This makes the product easily marketable as it offers the policyholder an unambiguous improvement (in a first-order stochastic dominance sense) over existing VA policies. Given the simplicity of implementation and the relative magnitude of the financial benefits to the innovative insurer, this simple one-time reduction of the VA fee rate appears to be a very attractive first step in the deviation from the current market status-quo, as demonstrated in Section 3.

As investors transition to the innovative VA provider, the other companies in the market will start to offer similar policies. In an effort to regain market share and increase overall profits, companies will want to further reduce ϕ_{red} to attract investors, at the expense of per-policy surplus. Eventually, providers may also alter the initial fee rate ϕ_{ini} (in addition to ϕ_{red} and n_{red}).

In this section we assume that the VA market is competitive on the provider side. This condition imposes a zero-profit restriction for the insurers. As a result, policies are "optimized" (over the fee structure $\hat{\phi}$) to provide the largest policy value (i.e. highest market value computed as the risk-neutral expected present value, V_0) to the investor, subject to the constraint that $NPV_0(\hat{\phi}) = 0$.

Relating the Fee Rates

In fact, for a given combination of ϕ_{red} and n_{red}—and taking into account the resulting optimal policyholder lapse-and-reentry behavior—the zero-profit restriction on the insurer uniquely specifies a value for ϕ_{ini}. This relationship is demonstrated in Figure 3.

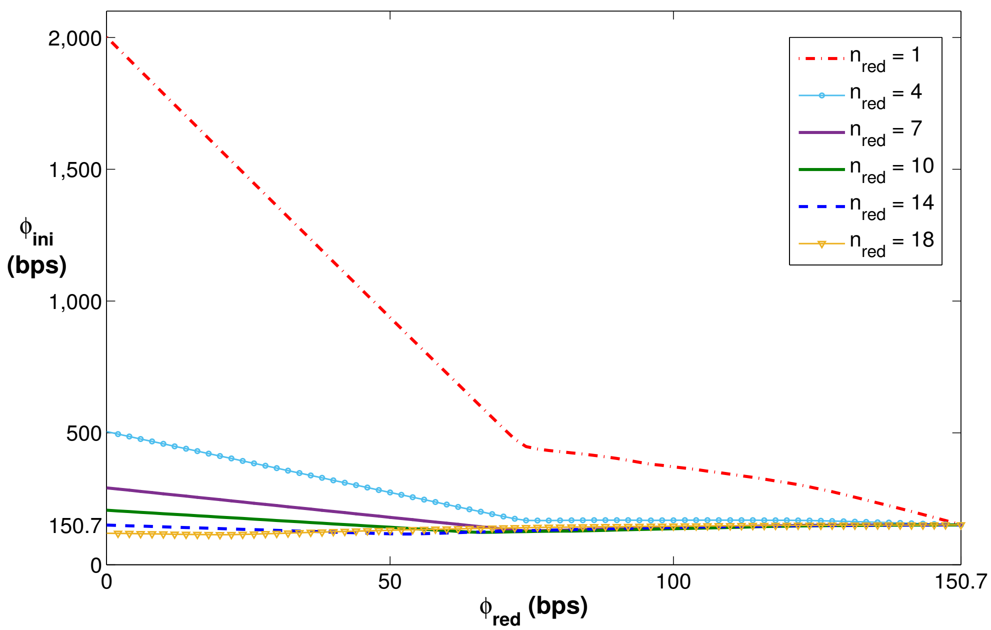
As before, when we impose a constant fee rate (that is, when ϕ_{red} is set to equal ϕ_{ini}), the break-even fee rate is approximately 150.7 bps. In most cases displayed in Figure 3—specifically, for $n_{\text{red}} \geqslant 7$— lowering ϕ_{red} (moderately) would allow the insurer to also reduce ϕ_{ini} (but to a lesser degree) while still breaking even in expectation. This observation is a consequence of a simultaneous reduction in the frequency of policy lapses, as shown in Figure 4. This graph also confirms our intuitive insight that lapsing becomes increasingly suboptimal as ϕ_{red} is reduced to 0. In particular—while L_0 may never be exactly equal to 0 in theory, as there always exist sufficiently large values of A_t for which it is optimal to lapse—we can define a threshold $\bar{\phi}_{\text{red}}$ as the largest value of ϕ_{red} for which there are almost no lapses:

$$\bar{\phi}_{\text{red}} \ := \ \max\{\phi_{\text{red}} \mid L_0 < 0.005\} \ .$$

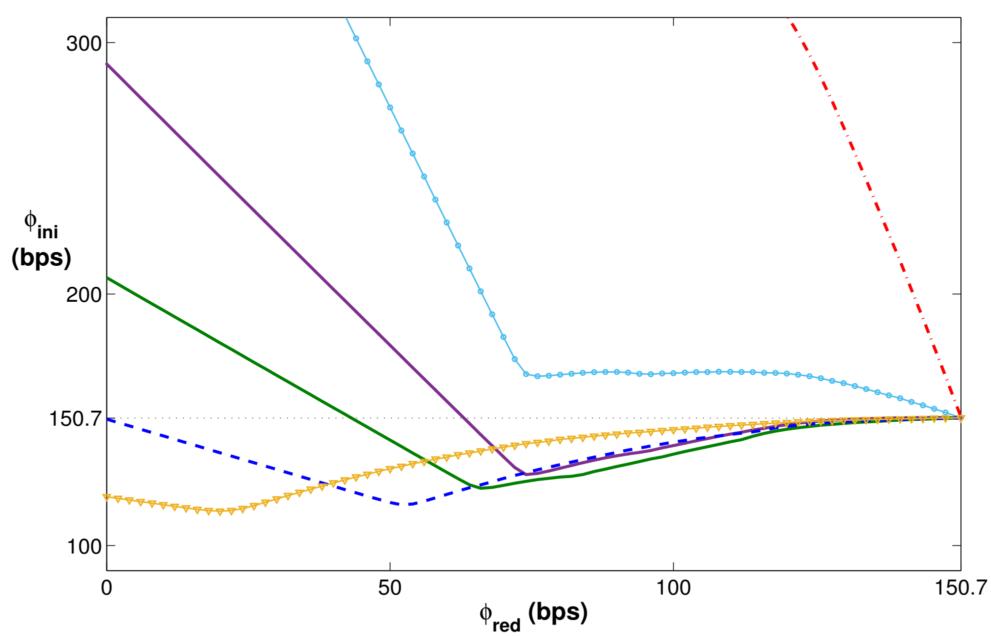
We find that under our model specifications such a threshold exists (that is, has a positive value) for every $n_{\text{red}} \leqslant 18$.

Here, as it was the case with an *innovative* VA provider, a reduction in the lapse rate (due to a reduced ϕ_{red}) lowers the insurer's overall policy acquisition expenses, and for moderate reductions in ϕ_{red} (that is, such that ϕ_{red} remains above $\bar{\phi}_{\text{red}}$), these savings evidently *exceed* the insurer's reduction in income caused by the lower fee rate. Contrary to our analysis in the previous section, and due to the zero-profit restriction imposed here on the insurer,

Figure 3: Relation of Break-Even Fee Rates (in bps) in Competitive Market.



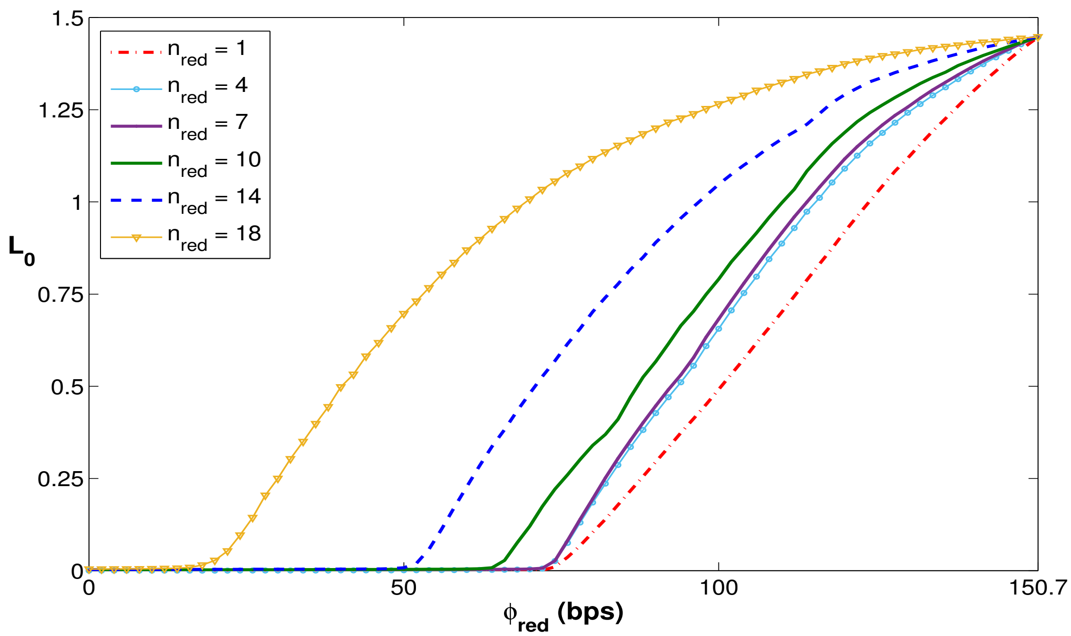
Same graph as above but with truncated y-axis (see legend from above graph):



Note: The insurer's net present value is calculated in expected present value terms under the measure $\mathbb{Q}$ (see Equation (8)) and is based on $\phi_{\text{ini}} = 150.7$ bps as well as on the parameters displayed in Table 1.

this net benefit is now also passed on to the policyholder, and manifests in the form of a lower initial fee rate ϕ_{ini}.[6] However, reducing ϕ_{red} below $\bar{\phi}_{\text{red}}$ has no additional effect on lapse behavior (and thus acquisition expenses), so that the insurer would need to *increase* ϕ_{ini} in order to compensate for the reduced income in the later policy years.

Figure 4: Average Number of Policy Lapses in a Competitive Market.



Note: Lapses are determined under $\mathbb{P}$ and are based on ϕ_{ini} from Figure 3.

Impact on Policy Value

In a competitive market environment where all "savings" are passed on to the policyholder, it is logical that the policy value is highest when expenses are lowest. As Figure 5 shows, for all values of n_{red} this occurs when $\phi_{\text{red}} = 0$ as the policy value is a strictly decreasing function of ϕ_{red}. Thereby, the graphs in Figure 5 consist of two parts. As discussed earlier,

[6] On the other hand, if the fee is reduced sooner—see the cases of $n_{\text{red}} = 1$ and $n_{\text{red}} = 4$ in Figure 3—the insurer's loss of fee income outweighs its gain from the reduced lapsing. As a result, ϕ_{ini} must be increased so that the insurer continues to break even in expectation.

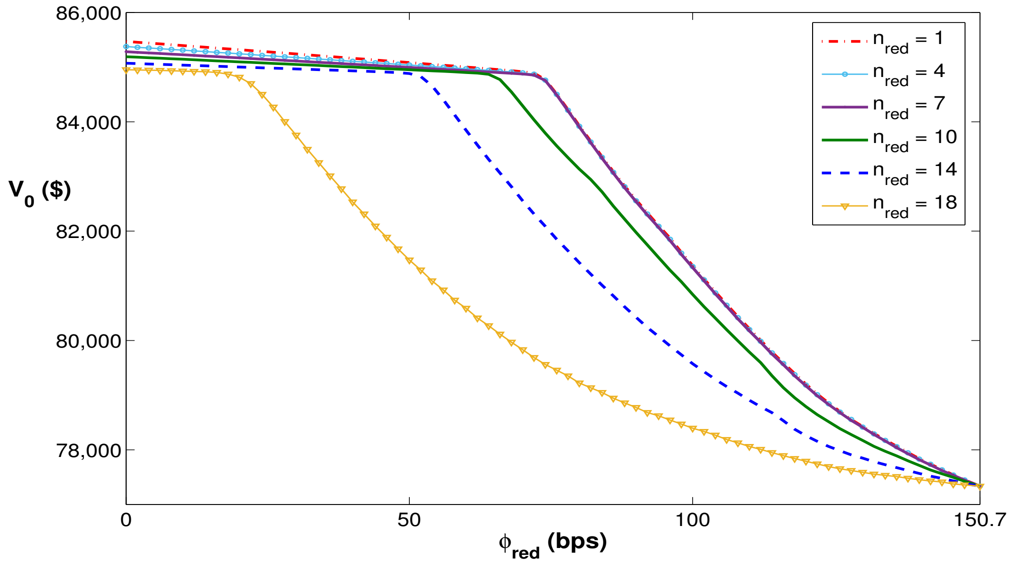
reducing ϕ_{red} (from the original 150.7 bps) leads to a reduction in the policy lapse rate, a resulting reduction in ϕ_{ini}, and thus a significant increase in the policy value V_0. This applies as we keep reducing ϕ_{red} until it reaches the threshold $\bar{\phi}_{\text{red}}$. From there on, the benefits to the policyholder from further reducing ϕ_{red} are significantly smaller, as they are largely offset by the necessary increase in ϕ_{ini} (see Figure 3). Nonetheless, to maximize the policy value, ϕ_{red} should be reduced all the way to 0.

The key insight here is that front-loading fee payments reduces the VA account value in the early years of the policy—while allowing the account value to catch up over time (due to lower fee rates in later policy years) compared to the case where the fee is spread out relatively more evenly over the policy period. Therefore, in the absence of lapses, at any given time the account value will be lower under the front-loaded free structure. Since in our model the insurer's expenses are assessed annually in proportion to the VA account value at the time, front-loading fees reduces the overall expense payments and thus increases the policy value. This explains why the policy value V_0 keeps increasing as ϕ_{red} is reduced below $\bar{\phi}_{\text{red}}$, and also suggests (accurately) that V_0 should be larger for lower values of n_{red}.

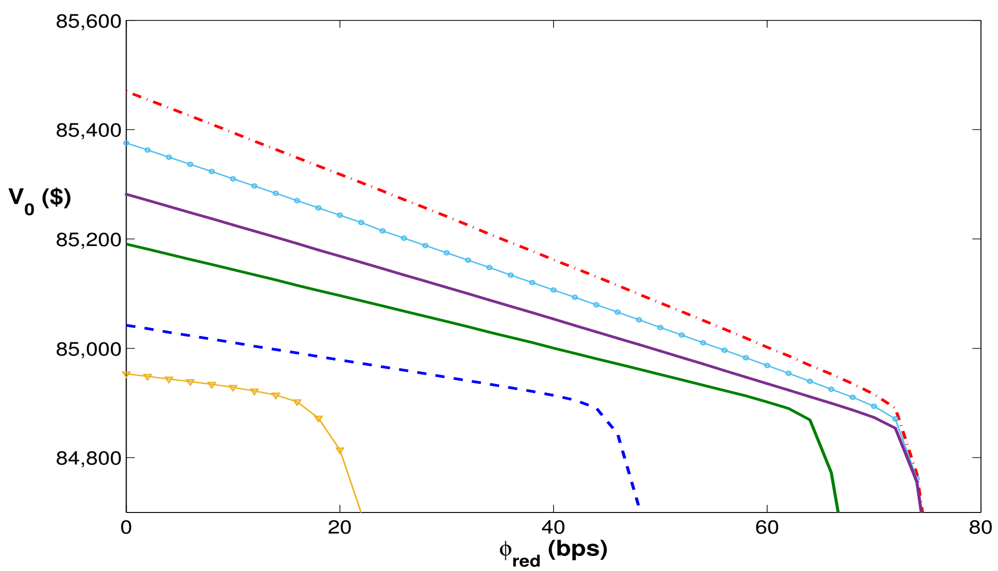
The valuation results displayed in Table 3 confirm these insights. Therefore, the theoretically optimal fee structure in a competitive market—based on our model specifications—is the one where $n_{\text{red}} = 1$ and $\phi_{\text{red}} = 0$, that is when all fees are front-loaded. This raises the policy value to \$85,470, which constitutes an increase of \$8,130 or 10.5% compared to the current status quo in the U.S. VA market.

However, this fee structure imposes a very large up-front fee on the policyholder (around 20% of the initial investment) and may therefore be optimal in theory but not necessarily from a marketing perspective. In particular—and aside from the emotional impact of agreeing to see 20% of your money vanish—can the average policyholder (or us without Matlab) tell whether this up-front fee structure is in any way preferable to the current 150.7 bps flat fee rate? Moreover, it is worth noting that the change in the policy value V_0 when $\phi_{\text{red}} < \bar{\phi}_{\text{red}}$ is driven only by the insurer's annually *recurring* expenses. Considering that in practice, some of these annually recurring expenses may actually be fixed rather

Figure 5: Policy Value in a Competitive Market.



Same graph as above but with truncated x-axis (see legend from above graph):



Note: The insurer's net present value is calculated in expected present value terms under the measure $\mathbb{Q}$ (see Equation (8)) and is based on $\phi_{\text{ini}} = 150.7$ bps as well as on the parameters displayed in Table 1.

Table 3: Optimal Fee Structure and Valuation Statistics for a Competitive Market.

	no red.	n_{red}					
		1	4	7	10	14	18
(a) Maximize V_0 s.t. $NPV_0 = 0$:							
ϕ^*_{ini} (bps)	150.7	2,001.2	504.8	291.5	206.5	150.3	119.4
ϕ^*_{red} (bps)	150.7	0.0	0.0	0.00	0.0	0.0	0.0
V_0 (\$)	77,340	85,470	85,380	85,280	85,190	85,070	84,950
$EPVE_0$ (\$)	22,660	14,530	14,620	14,720	14,810	14,930	15,050
L_0	1.45	0.00	0.00	0.00	0.00	0.00	0.00
(b) $\phi_{\text{red}} = \bar{\phi}_{\text{red}}$ and $NPV_0 = 0$:							
ϕ_{ini} (bps)	150.7	454.5	176.3	133.8	126.8	118.9	116.7
$\bar{\phi}_{\text{red}}$ (bps)	150.7	73.3	71.5	70.7	61.8	47.1	8.4
V_0 (\$)	77,340	84,840	84,880	84,870	84,890	84,910	84,930
$EPVE_0$ (\$)	22,660	15,160	15,120	15,130	15,110	15,090	15,070
L_0	1.45	0.00	0.00	0.00	0.00	0.00	0.00

Note: The table depicts fee rates and valuation statistics in a competitive market environment for various values of n_{red}, under two distinct constraints: (a) considers the optimal fee rates (ϕ_{ini}, ϕ_{red}) that maximize the policy value V_0, while (b) imposes that $\phi_{\text{red}} = \bar{\phi}_{\text{red}}$, the largest possible rate for the reduced fee such that the expected number of lapses $L_0 < 0.005$. In both cases, the competitive nature of this market implies the additional constraint that $NPV_0 = 0$, which uniquely determines the value of ϕ_{ini}(see Figure 3. The table also shows the expected expense payment $EPVE_0$. Results are based on the parameter values from Table 1. Lapses are assessed under the measure $\mathbb{P}$, all other values are computed under the measure $\mathbb{Q}$.

than proportional to the VA account value,[7] the policyholder would likely benefit even less from a reduction of ϕ_{red} below $\bar{\phi}_{\text{red}}$ than Figure 5 and Table 3 suggest. In contrast, this would have very little effect on the shape of the graphs in Figure 5 where $\phi_{\text{red}} > \bar{\phi}_{\text{red}}$, since there the change in the policy value is driven primarily by the drop in policy *acquisition* expenses resulting from a reduction in the frequency of lapses (as ϕ_{red} is reduced).

As a result, while front-loading all fees may be optimal in theory, policyholders can extract the vast majority of potential benefits from the VA with any fee structure that sufficiently dis-incentivizes lapsing. And as part (b) of Table 3 shows, such a product can have much more moderate fee rates. For instance, the policyholder may pay an initial fee rate of 133.8 bps, which is reduced to 70.7 bps after 7 years. Under this fee structure, the policy value is only \$600 (that is, around 0.7%) less—at most—than under complete front-loading, but \$7,530 (that is, around 9.7%) higher than under the current VA market status-quo. In addition, the fee is at all times significantly below the status-quo flat rate of 150.7 bps, which makes the benefits apparent to any policyholder.

5 Concluding Remarks

The goal of this paper is to explore the impact of the fee structure of VA contracts on the policyholders' decision to lapse. We find that a simple one-time fee reduction can offer sizeable disincentives to lapsing, and consequently makes the VA policy more interesting to both policyholder and insurer. For policyholders, the new design offers lower fee rates without compromising on any of the benefits and opportunities that make VAs an attractive investment in the first place. For VA providers, the proposed fee structure increases customer retention and considerably reduces acquisition expenses from "1035 exchanges." Furthermore, this one-time fee reduction is straightforward to implement and can be added simultaneously to new and *existing* VA policies. An innovative insurer would thus have immediate benefits. Therefore, we anticipate that our study could have a significant impact on the U.S. VA industry.

[7]Our model can easily be adjusted to account for the insurer's fixed expenses, namely by setting the "profit" (NPV_0) to equal these fixed expenses and reducing ε_{rec} accordingly.

Insurers can employ other VA policy features to mitigate lapse incentives. As noted for instance by Moenig and Zhu (2016), a ratchet-style guarantee and a state-dependent fee structure can be similarly effective in that regard. However, our proposed time-dependent fee structure may be preferable to either of these features for reasons beyond the consideration of our model. Indeed, one of the criticisms of the state-dependent fee (i.e. a fee that is reduced when the underlying fund is beyond some threshold) is that it only benefits policyholders whose investments have performed well, while policyholders with low account values will not have any reduced fees. In addition, the state-dependent fee design may lead to manipulation risk by creating incentives for insurers to keep the fund below some threshold. By contrast, the time-dependent fee structure that we are proposing is applied equally to *all* policyholders, regardless of investment performance. Also, a major disadvantage of ratchet-type guarantees in actuarial practice is that they are highly path-dependent and therefore difficult to hedge for insurers. Our proposed design, on the other hand, does not complicate the hedging of the guarantee. On the contrary, since frequent policy lapses interfere with the insurer's ability to hedge (see Kling, Ruez, and Ruß (2014)), the time-dependent fee structure could *improve* the hedging efficiency of the VA product.

By causing fewer policy lapses, the time-dependent fee structure would allow VA providers to increase their investment horizon. Gollier (2015) notes that it is beneficial to the prosperity of a country if pension funds and life insurers can invest in illiquid, long-term assets, and thereby contribute to the growth of the economy. This would also have additional benefits to the policyholder, as these assets offer a higher rate of return (see e.g. Huberman and Halka (2001); Browne, Milevsky, and Salisbury (2003); Ben-Rephael, Kadan, and Wohl (2015)); in addition, long-term stock investments benefit from a lower annual volatility due to the negative auto-correlation of stock returns (Campbell and Viceira, 2002; Bansal and Yaron, 2004). We leave the assessment and inclusion of these benefits for future research.

Finally, the model employed in this article considers VA policy exchanges in order to improve the value of the *GMDB* rider. However, we suspect that the insights provided here carry over to other VA guarantees, including e.g. lifetime withdrawal benefits. Indeed, the time-dependent fee applies during the accumulation period for guarantees that start after this accumulation period, which is common for most guarantees in VAs. Whether a time-dependent fee structure can indeed impact lapses in an equally significant way under more complex guarantees constitutes another interesting avenue for future research.

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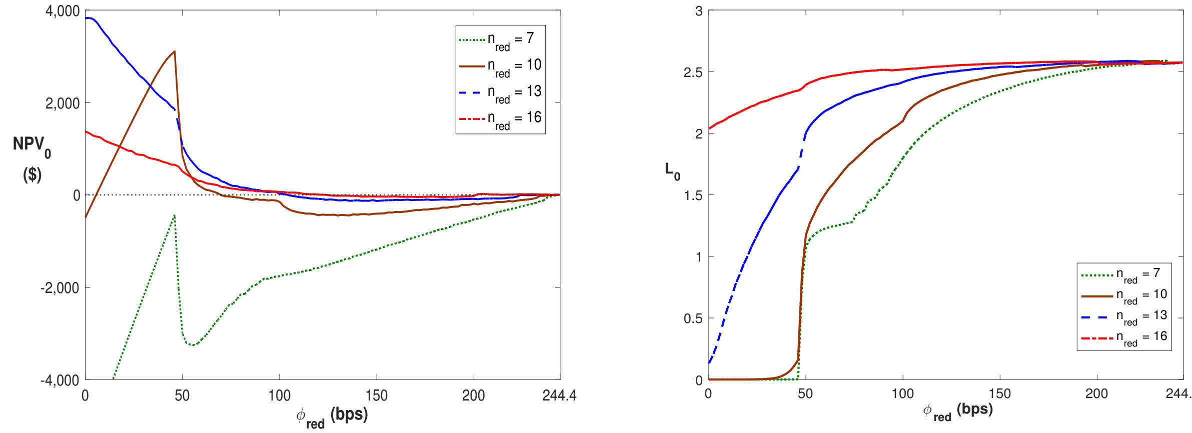
A Adding a Living Benefit Guarantee

As a robustness check for our proposed fee structure we consider the case where the insurer adds a living benefit guarantee to the VA+GMDB policy. For simplicity, we assume this to be a return-of-premium Guaranteed Minimum Accumulation Benefit (GMAB) rider. It ensures the policyholder that he will receive the larger of the VA account value and the guaranteed amount when the policy matures at time T—that is, $\max\{A_T, G_T\}$—provided that he is still alive at the time. Thereby, the guaranteed amount is the same as for the corresponding GMDB rider.

Figure 6: NPV and lapse rates with innovative insurer and living benefit guarantee.



(a) Net present value to insurer

(b) Average number of lapses

Note: NPV_0 is calculated in expected present value terms under the measure $\mathbb{Q}$ (see Equation (8)), and L_0 is calculated under the measure $\mathbb{P}$. Both quantities are based on $\phi_{\text{ini}} = 244.4$ bps as well as on the parameters displayed in Table 1 (with the exception that $\sigma = 15\%$).

We reflect the addition of the GMAB in the policyholder's optimization problem by replacing the terminal condition of Equation (6) with

$$V_T(A_T, G_T, m_T) = [1 - s(m_T)] \max\{A_T, G_T\} \, .$$

For our numerical implementation, we again rely on the parameter specifications of Table 1, with the exception of the investment volatility σ. In U.S. VAs it is common practice for insurers to restrict the policyholder's investment choices if she elects to add on a living benefit guarantee. The purpose is to limit the equity exposure of the VA account value in order to reduce the insurer's risk and thus also the value of the guarantee. Therefore, we assume a reduction in equity exposure to 75% of the benchmark case ($\sigma = 20\%$), so that the VA account volatility in the presence of the GMAB is $\sigma = 15\%$. Valuation and lapse statistics are displayed in Table 4 and Figure 6.

In particular, we observe that in the standard case (without fee reduction), the addition of

Table 4: Valuation and Lapse Statistics With Living Benefit Guarantee.

	no red.	n_{red}			
		7	10	13	16
ϕ^*_{red} (bps)	244.4	244.4	47.0	0	0
NPV_0^* (\$)	0	0	3,120	3,820	1,370
V_0 (\$)	77,490	77,490	82,060	81,620	78,930
$EPVE_0$ (\$)	22,510	22,510	14,820	14,560	19,700
L_0	2.58	2.58	0.19	0.13	2.04

Note: For select values of n_{red}, the table depicts the optimal reduced fee rate ϕ^*_{red} and corresponding maximum profit to the innovative insurer (NPV_0^*). It also shows the corresponding policy value V_0, expected expense payment $EPVE_0$, and average number of lapses L_0. Results are based on the parameter values from Table 1—although with $\sigma = 15\%$—and an initial fee rate $\phi_{\text{ini}} = 244.4$ bps. Lapses are assessed under the measure $\mathbb{P}$, all other values are computed under the measure $\mathbb{Q}$.

the GMAB leads to a significant increase in the annual break-even fee for the insurer—from 150.7 to 244.4 bps—despite the reduced volatility. This is because the guarantee may now be triggered more frequently, not only when the policyholder dies prematurely but also in case of his survival. This also drives up the policyholder?s incentive to lapse and reenter in order to increase the value of the guarantee (from 1.45 to 2.58 lapses per 25 policy years on average); this reflects in the higher fee rate as well. Interestingly, however, the present value of the insurer's overall expenses hardly changes (from \$22,660 to \$22,510). It appears that while the insurer has to pay policy acquisition expenses more frequently, the expense amounts per policy lapse (as well as the annually recurring expenses) are lower than without the GMAB. This is likely because expenses are assessed in proportion to the VA account value—which tends to be lower in the presence of the GMAB due to the increased fee rate.

Moreover, our earlier insights regarding the impact of a simple fee-reduction (here: from $\phi_{\text{ini}} = 244.4$ bps to ϕ_{red}, beginning after n_{red} contract years) extends to the GMAB rider, although results change on a quantitative level. For instance, we see from Figure 6(b) that in the case of $n_{\mathrm{red}} = 7$, the insurer can eliminate all lapse incentives if he reduces the fee rate far enough; however, Figure 6(b) shows that even in this case the lost fee income will outweigh his savings in expenses. Therefore, the insurer would not profit from reducing the fee rate this early into the contract. However, waiting just a few years longer with the fee reduction turns out to be beneficial to the VA provider. In particular, reducing the fee rate to around 30 bps after 10 years would eliminate all lapse incentives and allow the insurer to make a profit of around \$3,000. He can further improve his situation by waiting to time 13, and completely eliminating the fee at that time. This would result in the maximum profit of \$3,820. Conversely, if the fee reduction does not occur until later into the contract (see e.g. the results for the case of $n_{\mathrm{red}} = 16$), lapsing is still optimal in some occasions. Due to the resulting expenses, this fee strategy turns out to be less desirable for the VA provider. Overall, we conclude that our key insight extends to the case of living benefits as well, even though the larger required initial fee rate reduces the potential benefits that our proposed fee structure can provide.