# On the Interaction between the Demand for Saving and the Demand for Risk Reduction

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April 15, 2017

**Abstract**

This paper studies optimal saving decisions in the presence of an endogenous future consumption risk. The endogeneity arises because the decision-maker anticipates to engage in risk reduction, for example by purchasing insurance. We show that the interaction between saving and insurance is driven by whether absolute risk aversion decreases or increases in wealth, in which case insurance is a substitute or complement for saving as long as relative risk aversion is bounded by unity. Furthermore, for non-increasing absolute risk aversion saving is a substitute for insurance. These results carry over to more general forms of $n$th-degree risk reduction by formulating the conditions based on decreasing or increasing $n$th-degree Ross risk aversion instead. We also show that the endogeneity of future consumption risk is a critical determinant of the intensity of the precautionary saving motive conditional on preferences, so for a given degree of prudence.

**Keywords:** insurance · risk · risk aversion · saving

**JEL-Classification:** D11 · D14 · D81 · D91 · G22

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# 1 Introduction

It is common in the literature on optimal decision making under risk and uncertainty to focus on specific decision variables and to single them out when modeling the agent's cost-benefit trade-off. For instance, the propensities to purchase insurance, to engage in precautionary saving or to perform prevention activities are in many cases dealt with separately. There are several exceptions that illustrate that these types of decisions can interact in non-trivial ways, which is important both for normative as well as descriptive work and needs to be respected by the careful empiricist. Ehrlich and Becker's (1972) classical contribution was the first analysis dedicated to the interaction between various instruments - insurance and self-protection on the one hand, and insurance and self-insurance on the other hand - used to manage financial risks. This initiated a series of theoretical papers examining joint risk management decisions. Among these, Dionne and Eeckhoudt (1984) considered the relationship between saving and insurance decisions, Menegatti and Rebessi (2011) and Peter (2017) examined simultaneous saving and self-protection efforts and Courbage et al. (2015) study the interplay of preventive activities targeting different and potentially interdependent sources of risk.[1]

In this paper, we study the demand for (precautionary) saving when consumption risk is endogenous. This endogeneity arises when agents anticipate to engage in risk mitigation (e.g., insurance) in the future so that our contribution can be viewed as a direct extension of the stream of literature that analyzes the interaction of optimal decisions under risk. Taken in isolation, the demand for precautionary saving has been shown to depend - in the expected utility model - on the sign of the third derivative of the utility function (see Leland, 1968; Sandmo, 1970; Drèze and Modigliani, 1972) while Kimball (1990) indicated that the intensity of this demand was measured by the ratio of (minus) the third derivative to the second derivative of the utility function. In these contributions, the future risk is exogenous. Surprisingly, the presence of endogenous risks and their effect on saving has not been discussed much. This is despite the fact that individuals have a variety of instruments to address future consumption risks (insurance, self-protection, self-insurance, etc.). We thus consider in this paper that

[1] Note that the analysis of joint actions undertaken in order to protect oneself against disease has been introduced into the health economics literature in recent years, see for instance the analysis of self-protection activities and disease treatment in Hennessy (2008), Menegatti (2014) or Brianti et al. (2017).

individuals can engage in risk reduction at the point in time when they will be exposed to the risk so that the demand for saving and that for risk reduction interact. We consider the case of insurance on its own sake before moving to the more general case. Therefore, the first part of our analysis can be seen as a complement to Dionne and Eeckhoudt (1984) who study Hicksian demand for insurance and saving in a set-up where both decisions are taken in the first period. In our paper, we examine Walrasian demand and focus on the case where the decision-maker anticipates when taking his saving decision that future consumption risks will be mitigated via insurance in the period they occur.[2]

Our first result is that the interaction between saving and insurance depends on how the Arrow-Pratt measure of absolute risk aversion changes with wealth. Specifically, if relative risk aversion is bounded by unity an increase in the interest rate increases the demand for saving and decreases or increases the demand for insurance depending on whether absolute risk aversion is decreasing or increasing in wealth. So insurance is a substitute or complement for saving depending on the shape of the index of absolute risk aversion. Furthermore, in those cases where insurance is not a Giffen good and absolute risk aversion is non-increasing in wealth, an increase in the price of insurance decreases the demand for insurance (per definition, of course) and increases the demand for saving. In this case, saving is a substitute for insurance. We show that all of these results can be extended to general forms of $n$th-degree risk reduction by formulating the sufficient conditions in terms of how $n$th-degree Ross risk aversion depends on wealth. We also show that the endogeneity of risk influences the trade-off between consumption smoothing and precautionary saving. We present a numerical example that illustrates how precautionary saving as a fraction of the total demand for saving can vary between 0% and over 20% depending on the insurability of the consumption risk, even if the decision-maker's intensity of absolute prudence is at a constant level.

The paper is organized as follows. Section 2 presents a simple model of saving and insurance and analyzes the comparative statics of the joint optimum with respect to the interest rate and the price of insurance. Section 3 extends the analysis to a more general model of $n$th-degree risk reduction and the final section concludes.

[2] Hicksian demand is obtained by keeping expected utility constant in the comparative statics analysis in order to eliminate wealth effects and to isolate price effects.

# 2 A Simple Model of Saving and Insurance

## 2.1 Preliminaries

Consider a decision-maker (DM) who lives for two periods. His consumption stream $(c_1, \widetilde{c}_2)$ consists of certain consumption $c_1$ in the first period and risky consumption $\widetilde{c}_2$ in the second period, where the tilde indicates a random variable. The DM's discounted expected utility is given by $u(c_1) + \beta \mathbb{E} v(\widetilde{c}_2)$, where $u$ denotes the DM's first-period utility function, $v$ his second-period utility function, and $\beta$ the utility discount factor. We assume non-satiation and risk aversion in each period such that $u' > 0$, $u'' < 0$, $v' > 0$ and $v'' < 0$. The DM receives certain income of $w_1$ and $w_2$ in the first and the second period, respectively.

The uncertainty in the second period results from a potential loss of size $l$ that occurs with probability $p$. The DM's effective exposure to this risk is endogenous because we assume insurance to be available in the second period to protect against the financial consequences of a loss. We denote by $\alpha \in [0,1]$ the level of coverage and by $\lambda \geq 0$ the loading factor. Then, the per-unit price of insurance is given by $(1+\lambda)$ and the premium $\pi$ associated with a level of coverage of $\alpha$ is given by $(1+\lambda)\alpha p l$. Furthermore, the DM decides about his consumption allocation over time by specifying a level of saving $s$ in the first period. Savings are deducted from first-period income and yield interest according to the non-random interest rate $r \geq 0$ in the second period. With these specifications, the DM's objective function is given by

$$\max_{\alpha,s} U(\alpha,s) = u(w_1 - s) + \beta \left[ p v(w_2 + s(1+r) - \pi - (1-\alpha)l) + (1-p) v(w_2 + s(1+r) - \pi) \right].$$

Unlike previous literature, the saving decision in our model is undertaken in the presence of an *endogenous* additive consumption risk because the riskiness of the DM's second-period income depends on his insurance choice. To compress notation, we use subscripts 1, $2L$ and $2N$ to denote consumption in the first period, the second-period loss state and the second-period no-loss state, respectively. The first-order conditions for the DM's maximization problem are given by

$$\begin{aligned} U_\alpha &= \beta l \left[ (1-(1+\lambda)p) p v'_{2L} - (1+\lambda) p (1-p) v'_{2N} \right] = 0, \\ U_s &= -u'_1 + \beta(1+r) \left[ p v'_{2L} + (1-p) v'_{2N} \right] = 0. \end{aligned} \tag{1}$$

The first equation determines the optimal level of insurance coverage such that the marginal rate of substitution between second-period consumption in the loss and the no-loss state is equal to the slope of the line of insurance. The second equation specifies that expected marginal utility of consumption is equal over time at the optimal level of saving. We show in the appendix that the second-order conditions are satisfied.

Before we proceed, we inspect the cross-derivative of the DM's objective function with respect to the level of insurance and saving. Direct computation yields that

$$U_{\alpha s} = \beta(1+r)l\left[(1-(1+\lambda)p)pv''_{2L} - (1+\lambda)p(1-p)v''_{2N}\right],$$

which is sign ambiguous. If $A(w) = -v''(w)/v'(w)$ denotes the coefficient of Arrow-Pratt risk aversion of second-period utility function $v$, we can use the first-order condition for optimal insurance demand to rewrite the cross-derivative as follows:

$$U_{\alpha s} = \beta(1+r)(1+\lambda)p(1-p)lv'_{2N}\left[A_{2N} - A_{2L}\right].$$

This is informative about the relationship between insurance and saving at an optimal choice, which we summarize in the following remark.

**Remark 1.** *Insurance and saving are Edgeworth-Pareto substitutes (complements) in the sense of Samuelson (1974) when second-period risk aversion is decreasing (increasing) in wealth.*

A higher level of saving increases the individual's certain level of second-period consumption, which decreases the optimal demand for insurance when second-period utility exhibits DARA (see Mossin, 1968). Likewise, a higher level of insurance coverage reduces expected wealth due to the loading, which stimulates more savings, but also decreases the riskiness of second-period consumption which stimulates less savings (see Kimball, 1990). Under DARA, prudence is stronger than risk aversion so that the second effect preponderates and a lower level of saving will be optimal. Our discussion illustrates that there is a non-trivial interaction between the demands for saving and insurance as soon as the degree of risk aversion of second-period utility is not a constant function of wealth. This will be important in the comparative statics analysis.

## 2.2 Changes in the Interest Rate

In the presence of an exogenous income risk, it is well-known that optimal saving is increasing in the interest rate if relative risk aversion is less than unity (see Proposition 63 in Gollier, 2001). The DM trades off a substitution effect because a higher return on saving reinforces the incentive to save, against a wealth effect because a higher interest rate makes the DM wealthier in the second period, which attenuates the incentive to save. When risk is endogenous via insurance, we also need to determine how the DM's demand for insurance is affected by a change in the interest rate. We summarize our findings in the following proposition.[3]

**Proposition 1.** *Assume relative risk aversion of second-period utility to be bounded by unity. An increase in the interest rate increases the demand for saving and decreases (increases) the demand for insurance if second-period risk aversion is decreasing (increasing) in wealth.*

Said differently, insurance is a substitute (complement) for saving in the Walrasian sense when second-period risk aversion is decreasing (increasing) in wealth. The intuition behind this result is the following. An increase in the interest rate has a direct effect on each, saving and insurance, and the interaction between both instruments induces indirect effects (see Remark 1). For relative risk aversion below unity, the direct effect of a higher interest rate on saving is positive. The direct effect on insurance is a pure wealth effect, which is known to coincide with the sign of the slope of second-period risk aversion (see Mossin, 1968). This slope also governs the indirect effects. If second-period risk aversion is decreasing in wealth, the indirect effect of saving on insurance demand is negative and lowering the demand for insurance exerts a positive indirect effect on saving. Overall, direct and indirect effects are aligned so that the demand for saving increases and the demand for insurance decreases. Likewise, if second-period risk aversion is increasing in wealth, the indirect effect of saving on insurance demand is positive and increasing the demand for insurance exerts a positive indirect effect on saving. Again, direct and indirect effects are aligned so that both the demand for saving and the demand for insurance increase.

[3] All proofs are gathered in the appendix.

Our result shows that Remark 1 about the substitution effect between saving and insurance generalizes to the optimal demands for both activities when we add the condition that ensures that saving reacts positively to an increase of the interest rate. Dionne and Eeckhoudt (1984) show that under decreasing temporal risk aversion, saving and insurance are substitutes in the Hicksian sense, so for price changes that keep expected utility constant. Our finding extends this result to the case where the insurance premium is paid in the second period and Walrasian instead of Hicksian demand is considered. We show further that insurance can be a complement for saving when second-period risk aversion is increasing in wealth.

### 2.3 Changes in the Price of Insurance

As was first discussed by Hoy and Robson (1981), insurance can be a Giffen good in the standard model of insurance demand. The reason is that a price increase will induce a positive wealth effect on insurance demand as soon as utility is DARA. Hoy and Robson (1981) and later Briys et al. (1989) and Hau (2008) discuss necessary and/or sufficient conditions for insurance not to be Giffen. As it turns out, insurance will not be Giffen in most circumstances so that we exclude this possibility in our further analysis. We summarize our findings in the following proposition.

**Proposition 2.** *Assume insurance not to be Giffen and second-period risk aversion to be non-increasing in wealth. Then, an increase in the price of insurance decreases the demand for insurance and increases the demand for saving.*

This result shows that saving is a substitute for insurance when second-period risk aversion is non-increasing in wealth. The intuition is similar to before. An increase in the price of insurance has a negative direct effect on the demand for insurance and a positive direct effect on the demand for saving. The former comes from the fact that we exclude the cases in which insurance might be Giffen, the latter results from the reduction in expected second-period income induced by the price increase. It induces the DM to save more to smooth consumption over the lifecycle. Indirect effects are aligned with these direct effects as long as second-period risk aversion is not increasing in wealth (see Remark 1).

Notice that unlike in the previous proposition, saving will not necessarily be a complement for insurance when second-period risk aversion is increasing in wealth. The reason is that the direct effect of an increase in the price of insurance on saving is positive due to risk aversion alone, independent of whether second-period risk aversion is increasing or decreasing in wealth. This results from the DM's propensity to smooth consumption over the lifecycle. As a result the direct and the indirect effect of an increase in the price of insurance on saving are conflicting in such a situation and the net effect depends on their relative strength.

## 2.4 Consumption Smoothing and Precautionary Saving

Thus far we have shown that under reasonable technical conditions, insurance is a substitute for saving and vice versa. To analyze the effect of the insurability of the consumption risk at the margin, we will start with the case of an exogenous risk and carve out the DM's saving response, both in terms of consumption smoothing and precautionary saving, when the risk becomes just insurable.

The case of an exogenous risk is a special case of our analysis for $\alpha^* = 0$. Let $s_0$ denote the optimal level of saving in the absence of insurance, which is determined by

$$u'(w_1 - s_0) = \beta(1+r)\left[pv'(w_2 + s_0(1+r) - l) + (1-p)v'(w_2 + s_0(1+r))\right].$$

These choices, $\alpha^* = 0$ and $s^* = s_0$, turn out to be the optimal decisions for any loading factor exceeding what we call the critical loading factor,

$$\lambda^{crit} = \frac{(1-p)\left[v'(w_{2L}^0) - v'(w_{2N}^0)\right]}{pv'(w_{2L}^0) + (1-p)v'(w_{2N}^0)}.$$

$w_{2L}^0$ and $w_{2N}^0$ are shorthand for the consumption levels $w_2 + s_0(1+r) - l$ and $w_2 + s_0(1+r)$. The DM pursues saving for two purposes, consumption smoothing and precaution. To disentangle the two, we denote by $s_0^{cs}$ the DM's saving choice if second-period income was risk-free:

$$u'(w_1 - s_0^{cs}) = \beta(1+r)v'(w_2 + s_0^{cs}(1+r) - pl).$$

As first shown by Leland (1968) and Sandmo (1970) and more systematically by Kimball

(1990), the DM engages in precautionary saving, $s_0 > s_0^{cs}$, if and only if he is prudent ($v''' > 0$). Our next result shows that the endogeneity of the second-period consumption risk can tilt the balance between consumption smoothing and precautionary saving towards the former.

**Proposition 3.** *Assume that second-period risk aversion is non-increasing in wealth. At the margin, the insurability of risk in the second period increases the demand for saving to smooth consumption and reduces the demand for precautionary saving.*

The intuition behind this result is similar to before. At the margin, the DM will start utilizing insurance as soon as the loading factor drops below the critical loading factor. The use of insurance results in a decrease in expected wealth as well as a decrease in risk in the second period. The first effect reinforces saving to smooth consumption whereas the second one reduces the DM's propensity to save for precautionary purposes. Due to the fact that the overall demand for saving decreases (see Proposition 2), the first effect reinforces the second one and the precautionary demand for saving is diminished. This finding is important because it highlights that the demand for saving and for precautionary saving depend critically on insurance market conditions via the endogeneity of risk. As a result, variations in these conditions will induce variations in optimal saving choices even if the underlying risk and time preferences of the agents are identical. Obviously, such considerations bear significant empirical measurement ramifications.

### 2.5 A Numerical Example

The following example serves to illustrate our findings about the substitution between saving and insurance and about the effects of insurability on the trade-off between consumption smoothing and precautionary saving. We consider an individual with $u(w) = v(w) = \log(w)$ and $\beta = 0.99$, who earns risk-free income of $w_1 = w_2 = \$50,000$ in each period and is subject to a 10% chance of a $10,000 loss in the second period. The following table summarizes the optimal demand for insurance and saving as a function of the interest rate, when the loading factor is given by 10%.

| $r$ | 0% | 1% | 2% | 3% | 4% | 5% |
|---|---|---|---|---|---|---|
| $\alpha^*$ | 49.74% | 49.49% | 49.23% | 48.98% | 48.73% | 48.47% |
| $s^*$ | \$301.51 | \$544.80 | \$783.33 | \$1,017.22 | \$1,246.62 | \$1,471.65 |

Consistent with Proposition 1, we observe that insurance is a substitute for saving because as the demand for saving increases, the demand for insurance decreases. In this particular example, the effect of the interest rate on insurance demand turns out to be marginal, whereas the effect of the interest rate on optimal savings is quite sizeable. The next table illustrates Propositions 2 and 3. The interest rate is fixed at 1% and the loading varies in increments of 5%. The last two columns also state the portion of saving that arises from consumption smoothing ($s_{cs}$) and the difference between the total demand for saving and the level of saving from consumption smoothing ($s^* - s_{cs}$), so in other words the precautionary demand for saving.

| $\lambda$ | 0% | 5% | 10% | 15% | 20% | $\lambda^{crit}$ |
|---|---|---|---|---|---|---|
| $\alpha^*$ | 100% | 73.68% | 49.49% | 27.15% | 6.44% | 0% |
| $s^*$ | \$495.05 | \$519.93 | \$544.80 | \$569.68 | \$594.55 | \$602.67 |
| $s_{cs}$ | \$495.05 | \$513.38 | \$519.67 | \$515.31 | \$501.46 | \$495.05 |
| $s^* - s_{cs}$ | \$0 | \$6.55 | \$25.13 | \$54.37 | \$93.09 | \$107.62 |

Consistent with Proposition 2, we observe that saving is a substitute for insurance because as the demand for insurance decreases, the demand for saving increases. As shown in Proposition 3, when the risk of loss becomes just insurable (i.e., considering a marginal decrease of $\lambda$ starting at $\lambda^{crit}$), saving to smooth consumption increases whereas the total demand for saving decreases so that the precautionary demand for saving decreases. Saving to smooth consumption is hump-shaped because the expected cost of the risk of loss in the second period (i.e., the insurance premium at the optimal insurance choice plus the expected loss cost) is hump-shaped, too. Notice that although the DM's relative prudence is given by 2 in all scenarios the intensity of precautionary saving varies with the price of insurance from 21.74% if the risk is exogenous ($\lambda = \lambda^{crit}$) to 0% if insurance is actuarially fair and full coverage is purchased.

# 3 A More General Model of Saving with Endogenous Risk

## 3.1 Preliminaries

Insurance is a costly activity that reduces the risk of second-period consumption in the sense of Rothschild and Stiglitz (1970). Indeed, for a given level of saving, we can rewrite second-period consumption levels in the loss and the no-loss state as follows:

$$\begin{aligned} w_{2L} &= w_2 + s(1+r) - \lambda\alpha pl - (1-\alpha(1-p))l, \text{ and} \\ w_{2N} &= w_2 + s(1+r) - \lambda\alpha pl - \alpha pl. \end{aligned}$$

Obviously, an increase in the level of coverage comes at a cost, which we can identify as $\lambda\alpha pl$, in the above equations. It is the portion of the premium that is in excess of the actuarially fair premium and it is increasing in the level of coverage. The remainder of the change is a mean-preserving contraction in the second-period wealth distribution because apparently $p \cdot (1-\alpha(1-p))l + (1-p) \cdot \alpha pl = pl$, which is the expected loss.

Inspired by this decomposition, we can analyze the interaction between saving and costly changes in risk more generally. To describe the effects of the risk-reducing activity performed in the second period, suppose that $F$ and $G$ are two cumulative distribution functions defined within the interval $[a,b]$. In what follows, $F_0(w)$ denotes the density function, $F_1(w)$ the cumulative distribution function, and more generally, $F_n(w) = \int_a^x F_{n-1}(z)\mathrm{d}z$ for $x \in [a,b]$ and $n \geq 1$. We recall the following definition based on Ekern (1980).

**Definition 1.** *The distribution $F$ has more $n$th-degree risk than $G$, $G \succ_n F$, if*

*(i) $G_k(b) = F_k(b)$ for $k = 1, 2, \ldots, n$,*

*(ii) $G_n(w) \leq F_n(w)$ for all $w \in [a,b]$ with strong inequality for some $w$.*

The first condition is necessary and sufficient for the $(n-1)$ first moments of $G$ and $F$ to coincide whereas the second condition is sufficient (but not necessary) for the $n$th moment of $F$ sign adjusted by $(-1)^n$ to exceed the $n$th moment of $G$ sign adjusted by $(-1)^n$. In the expected utility model, preferences over $n$th-degree changes in risk in the sense of Ekern (1980) are identified by the signs of subsequent derivatives of the utility function, which motivates the following definition.

**Definition 2.** *An agent is nth-degree risk-averse if* sgn $v^{(n)}(w) = (-1)^{n+1}$.

The link between $n$th-degree risk changes and $n$th-degree risk aversion is formulated in the following theorem.

**Theorem 1.** *The following two statements are equivalent:*

*(i)* $G \succ_n F$,

*(ii)* $\int_a^b v(w)\mathrm{d}G(w) > \int_a^b v(w)\mathrm{d}F(w)$, *for all functions $v$ such that* sgn $v^{(n)}(w) = (-1)^{n+1}$.

We follow Jindapon and Neilson (2007) to model the DM's opportunity to reduce the level of $n$th-degree risk of her second-period wealth distribution through a costly activity, whose intensity is denoted by $t$. Formally, a given level of the risk-reducing activity induces the wealth distribution $H_1(w,t) = (1-t)F_1(w) + tG_1(w)$ and the unit cost of the activity is denoted by $c > 0$. Our discussion at the beginning of this subsection makes it clear that insurance is a special case of such an activity.

Under these assumptions, the DM's objective function is given by

$$\max_{t,s} \left\{ U(t,s) = u(w_1 - s) + \beta \int_a^b v(w + s(1+r) - ct)\mathrm{d}H_1(w,t) \right\}.$$

To compress notation, let $w + s(1+r) - ct$ be denoted by $z(w)$. The first-order conditions related to the DM's problem are:

$$\begin{aligned} U_t &= -\beta c \int_a^b v'(z(w))\mathrm{d}H_0(w,t) + \beta \int_a^b v(z(w))\mathrm{d}\left[G_1(w) - F_1(w)\right] = 0, \\ U_s &= -u'(w_1 - s) + \beta(1+r) \int_a^b v'(z(w))\mathrm{d}H_1(w,t) = 0. \end{aligned}$$

The first equation determines the optimal level of the risk-reducing activity in terms of marginal benefit and marginal cost. The second equation specified the optimal level of saving by equating expected marginal utility over the lifecycle. As in Jindapon and Neilson (2007), we assume the objective function to be concave in the choice variables such that the system of first-order conditions identifies a maximum.

Via integration by parts, the first-order condition $U_t = 0$ can be rewritten as

$$U_t = -\beta c \int_a^b v'(z(w)) \mathrm{d}H_0(w,t) - \beta(-1)^n \int_a^b v^{(n)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w = 0$$

or equivalently as

$$-\frac{(-1)^n \int_a^b v^{(n)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w}{\int_a^b v'(z(w)) \mathrm{d}H_0(w,t)} = c. \tag{2}$$

As demonstrated by Jindapon and Neilson (2007), the choice of $t$ is not ranked according to Arrow-Pratt risk aversion but according to Ross risk aversion (see their Theorem 2). As such it is not surprising that the interaction between risk reduction and saving is governed by the effect of changes in wealth on Ross risk aversion. We provide the following definition, which is obtained by the characterization in Proposition 2.5 in Wang and Li (2014) and interpreted in the strict sense.

**Definition 3.** *The utility function $v$ displays decreasing $n$th-degree Ross risk aversion ($n \geq 2$) if for any $x$ and $y$, there exists a scalar $\lambda_n$ such that*

$$-\frac{v^{(n+1)}(x)}{v^{(n)}(x)} > \lambda_n > -\frac{v''(y)}{v'(y)}. \tag{3}$$

If the inequalities are reversed, we speak of increasing $n$th-degree Ross risk aversion and if the left hand side and the right hand side coincide for any $x, y$ we speak of constant $n$th-degree Ross risk aversion. The cross-derivative of the DM's objective function with respect to the intensity of the risk-reducing activity and saving is given by

$$U_{ts} = -\beta(1+r)c \int_a^b v''(z(w)) \mathrm{d}H_0(w,t) - \beta(1+r)(-1)^n \int_a^b v^{(n+1)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w. \tag{4}$$

To understand when there is a substitution effect between both activities, we rewrite $U_{ts} < 0$ with the help of equation (2) as follows:

$$\frac{(-1)^n \int_a^b v^{(n+1)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w}{(-1)^n \int_a^b v^{(n)}(z(w)) \left[F_n(w) - G_n(w)\right] \mathrm{d}w} < \frac{\int_a^b v''(z(w)) \mathrm{d}H_0(w,t)}{\int_a^b v'(z(w)) \mathrm{d}H_0(w,t)}. \tag{5}$$

If $v$ exhibits decreasing $n$th-degree Ross risk aversion, we obtain from equation (3) that

$$-(-1)^n v^{(n+1)}(x)v'(y) < -(-1)^n v^{(n)}(x)v''(y) \quad \forall x, y,$$

which implies that inequality (5) is satisfied. On the contrary, if utility function $v$ displays increasing $n$th-degree Ross risk aversion, the reverse inequality holds such that $U_{ts} > 0$. We summarize this in the following remark.

**Remark 2.** *$n$th-degree risk reduction and saving are Edgeworth-Pareto substitutes (complements) in the sense of Samuelson (1974) when second period $n$th-degree Ross risk aversion is decreasing (increasing) in wealth.*

The intuition is similar to before and this finding will allow us to sign the indirect effects between risk reduction and saving in the subsequent analysis.

### 3.2 Changes in the Interest Rate

We first wonder how the joint demand for saving and risk reduction is affected by changes in the interest rate. As it turns out, we can recover a version of Proposition 1 for the general case of costly $n$th-degree risk reduction activities. Our result is summarized in the following proposition.

**Proposition 4.** *Assume relative risk aversion of second-period utility to be bounded by unity. An increase in the interest rate increases the demand for saving and decreases (increases) the demand for $n$th-degree risk reduction if second-period $n$th-degree Ross risk aversion is decreasing (increasing) in wealth.*

This result shows that $n$th-degree risk reduction is a substitute (complement) for saving in the sense of Walrasian demand functions when second-period $n$th-degree Ross risk aversion is decreasing (increasing) in wealth. Assume that the interest rate increases. The direct effect on saving is positive as long as relative risk aversion is bounded by unity. Furthermore, this increase in the interest rate increases the individual's wealth in the second period, which reduces his propensity to invest in $n$th-degree risk reduction due to decreasing $n$th-degree Ross risk aversion. So the direct effect on the demand for risk reduction is negative. The indirect effects are governed by the substitution effect between saving and risk reduction, see Remark 2.

### 3.3 Changes in the Price of Risk Reduction

Similarly, we can conduct comparative statics with respect to $c$, the per-unit price of $n$th-degree risk reduction. As in the case of insurance, the demand for risk reduction can be Giffen, which is a possibility that we exclude from our analysis.[4] Under these presuppositions, we obtain the following result.

**Proposition 5.** *Assume $n$th-degree risk reduction not to be Giffen and $n$th-degree Ross risk aversion to be non-increasing in wealth. Then, an increase in the per-unit price of risk reduction decreases the demand for risk reduction and increases the demand for saving.*

The proof is very similar to the previous proofs and is omitted. If the per-unit price of risk reduction increases, this exerts a negative direct effect on the demand for risk reduction per assumption. Also, wealth in the second period is reduced due to the higher cost of risk reduction which exerts a positive effect on the demand for saving. This effect results from the concavity of utility. To achieve consistency between the direct and the indirect effects, $n$th-degree Ross risk aversion needs to be decreasing in wealth or at least not increasing in wealth so that the direct effect that less risk reduction exerts on saving is positive and the direct effect that higher savings exert on risk reduction is negative.

## 4 Conclusion

Many papers have been written on the analysis of one specific decision under risk and uncertainty. This is the case for precautionary saving which is defined as the extra saving due to risky future income. In the literature, the risk in question is usually considered as a background risk since it is assumed that it cannot be modified through preventive actions, diversified or insured against. These paper thus deal with the way the introduction of an exogenous risk affects savings. Less attention has been dedicated to the interaction between

[4] A sufficient condition for $n$th-degree risk reduction not to be Giffen is that $n$th-degree relative risk aversion be bounded by $n$ and that total spending on risk reduction as a fraction of wealth be bounded by $1/(n+1)$.

saving and other economic decisions that could be used to deal with a future risk, which would then be endogenous. This is the question we address in this paper since we analyze the relationship between the demand for saving and insurance or between saving and risk reduction more generally.

In the case of insurance, we find that the way the Arrow-Pratt coefficient of absolute risk aversion changes in wealth determines the interaction between the demand for saving and the demand for insurance. As a result, in those cases where saving reacts positively to an increase in the interest rate, insurance is a substitute of complement for saving whenever absolute risk aversion is decreasing or increasing in wealth, respectively. Similarly, if we exclude insurance to be Giffen, saving is a substitute for insurance whenever absolute risk aversion is non-increasing in wealth. As a consequence, in the more plausible case of decreasing absolute risk aversion, insurance and saving are substitutes for each other. We also show that this result extends to more forms of costly $n$th-degree risk reduction by drawing on the notion of decreasing $n$th-degree Ross risk aversion instead of Arrow-Pratt risk aversion.

We believe that these findings are interesting on their own behalf. However, as we illustrate for the case of saving and insurance, they have very direct empirical measurement implications and need to be taken into account when assessing the strength of the precautionary saving motive or inferring the intensity of certain preferences conditions from it. We illustrate that the insurability of risk or, more generally, the market conditions for risk-reducing activities are directly reflected in the extent to which decision-makers engage in precautionary saving. As a consequence, prudent decision-makers may not engage in precautionary saving when they anticipate sufficient risk reduction activities in the future, or they may engage in substantial precautionary saving when they anticipate the opposite. It becomes clear that their *beliefs* about future risk reduction opportunities and their associated costs will critically modulate the amount of precautionary savings. Our paper formalizes the exact mechanism behind this interaction and provides guidance for more informed identification and estimation.

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# Appendix

## A.1 Second-order conditions for system (1)

Direct computation shows that

$$\begin{aligned} U_{\alpha\alpha} &= \beta l^2 \left[(1-(1+\lambda)p)^2 p v''_{2L} + (1+\lambda)^2 p^2 (1-p) v''_{2N}\right] < 0, \quad \text{and} \\ U_{ss} &= u''_1 + \beta(1+r)^2 \left[p v''_{2L} + (1-p) v''_{2N}\right] < 0. \end{aligned}$$

After some simplifications, the determinant of the Hessian matrix of $U(\alpha, s)$ can be shown to take the following form:

$$D = U_{\alpha\alpha}U_{ss} - U_{\alpha s}^2 = u''_1 U_{\alpha\alpha} + \beta^2 (1+r)^2 l^2 p(1-p)(1-2(1+\lambda)p)^2 v''_{2L} v''_{2N} > 0.$$

As a result, the DM's objective function is globally concave in $(\alpha, s)$.

## A.2 Proof of Proposition 1

We utilize the two-dimensional Implicit Function Theorem and obtain that

$$\frac{\mathrm{d}\alpha}{\mathrm{d}r} = \frac{1}{D}\left(-U_{ss}U_{\alpha r} + U_{\alpha s}U_{sr}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}r} = \frac{1}{D}\left(-U_{\alpha\alpha}U_{sr} + U_{\alpha s}U_{\alpha r}\right).$$

To sign these two expressions, we need to determine the two remaining cross-derivatives $U_{\alpha r}$ and $U_{sr}$. Direct computation shows that

$$U_{\alpha r} = \beta l s \left[(1-(1+\lambda)p) p v'_{2L} - (1+\lambda)p(1-p) v''_{2L}\right] = \beta l s (1+\lambda) p (1-p) v'_{2N} \left[A_{2N} - A_{2L}\right],$$

where the second equality is obtained by using $U_\alpha = 0$. As a result, $U_{\alpha r}$ is negative (positive) whenever risk aversion of second-period utility is decreasing (increasing) in wealth. Per Remark 1 this shows that $U_{\alpha r}$ and $U_{\alpha s}$ coincide in sign. Furthermore,

$$\begin{aligned} U_{sr} &= \beta\left[p v'_{2L} + (1-p) v'_{2N}\right] + \beta(1+r)s\left[p v''_{2L} + (1-p) v''_{2N}\right] \\ &= \beta\left\{p v'_{2L}\left[1 + (1+r)s\frac{v''_{2L}}{v'_{2L}}\right] + (1-p) v'_{2N}\left[1 + (1+r)s\frac{v''_{2N}}{v'_{2N}}\right]\right\}. \end{aligned}$$

where the square brackets compare partial risk aversion in the second-period loss state and the second-period no-loss state with unity. According to Lemma 2 in Chiu et al. (2012), partial risk aversion is uniformly less than unity if and only if relative risk aversion is, in which case $U_{sr}$ is positive. This completes the proof.

## A.3 Proof of Proposition 2

Another application of the Implicit Function Theorem shows that

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\lambda} = \frac{1}{D}\left(-U_{ss}U_{\alpha\lambda} + U_{\alpha s}U_{s\lambda}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}\lambda} = \frac{1}{D}\left(-U_{\alpha\alpha}U_{s\lambda} + U_{\alpha s}U_{\alpha\lambda}\right).$$

To sign these expressions, the two missing cross-derivatives $U_{\alpha\lambda}$ and $U_{s\lambda}$ will be determined in the sequel. We obtain that

$$U_{\alpha\lambda} = -\beta l\left[p^2 v'_{2L} + (1-p)pv'_{2N}\right] - \beta\alpha pl^2\left[(1-(1+\lambda)p)pv''_{2L} - (1+\lambda)p(1-p)v''_{2N}\right].$$

Its sign is a priori ambiguous but we exclude insurance demand to be Giffen so that $U_{\alpha\lambda} \leq 0$.[5] The cross-derivative of the objective function with respect to saving and the price of insurance is

$$U_{s\lambda} = -\alpha pl\beta(1+r)\left[pv''_{2L} + (1-p)v''_{2N}\right],$$

which is positive due to risk aversion in the second period. This concludes the proof.

## A.4 Proof of Proposition 3

For $\lambda = \lambda^{crit}$, we know that $\alpha^* = 0$ is optimal. Therefore, $U_{s\lambda} = 0$ when evaluated at $\lambda^{crit}$ so that the effect of insurability on the total demand for saving reduces to

$$\left.\frac{\mathrm{d}s_0}{\mathrm{d}\lambda}\right|_{\lambda=\lambda^{crit}} = \frac{1}{D}U_{\alpha s}U_{\alpha\lambda}.$$

[5] Exploiting the first-order condition for optimal insurance demand, a sufficient condition for $U_{\alpha\lambda} \leq 0$ is that

$$A_{2L} - A_{2N} \leq \frac{1}{\alpha pl(1-p)(1+\lambda)}.$$

Loosely speaking, this is more likely to be satisfied if second-period risk aversion does not decrease too quickly in wealth and/or if the loading factor is not excessive.

Furthermore, at $\lambda = \lambda^{crit}$ the effect of the price of insurance on insurance demand reduces to $U_{\alpha\lambda}|_{\lambda=\lambda^{crit}} = -\beta l \left[p^2 v'_{2L} + (1-p)pv'_{2N}\right] < 0$. To investigate the demand for saving to smooth consumption, we start with an arbitrary loading factor, $\lambda \in [0, \lambda^{crit})$, and associated insurance demand of $\alpha = \alpha^*$. Expected wealth in the second period is given by

$$\overline{w}_2 = w_2 + s(1+r) - (1+\lambda)\alpha^* pl - (1-\alpha^*)pl$$

and the demand for saving to smooth consumption over time is implicitly defined via

$$-u'(w_1 - s_0^{cs}) + \beta(1+r)v'(\overline{w}_2) = 0.$$

The effect of a change in the price of insurance is obtained from the Implicit Function Rule:

$$-\beta(1+r)v''(\overline{w}_2)\alpha^* pl - \beta(1+r)v''(\overline{w}_2)\frac{\mathrm{d}\alpha^*}{\mathrm{d}\lambda}\lambda pl.$$

The first term is zero when evaluated at $\lambda = \lambda^{crit}$ due to $\alpha^* = 0$, and as a result

$$\left.\frac{\mathrm{d}s_0^{cs}}{\mathrm{d}\lambda}\right|_{\lambda=\lambda^{crit}} < 0.$$

It follows that a marginal decrease of $\lambda$ starting at $\lambda^{crit}$ strictly increases the level of saving to smooth consumption whereas the total level of saving remains constant or decreases, depending on whether second-period risk aversion is constant or decreasing. As a consequence, the demand for precautionary saving decreases.

### A.5 Proof of Proposition 4

We utilize the two-dimensional Implicit Function Theorem and obtain that

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \frac{1}{D}\left(-U_{ss}U_{tr} + U_{ts}U_{sr}\right) \quad \text{and} \quad \frac{\mathrm{d}s}{\mathrm{d}r} = \frac{1}{D}\left(-U_{tt}U_{sr} + U_{ts}U_{tr}\right).$$

where $D$ denotes the determinant of the Hessian of $U$. It holds that

$$U_{ss} = u''(w_1 - s) + \beta(1+r)^2 \int_a^b v''(z(w))\mathrm{d}H_1(w,t) < 0$$

due to concavity of $u$ and $v$. For the determinant $D$ of $U$ to be positive for maximality, it follows that $U_{tt} < U_{ts}^2/U_{ss} < 0$. Direct computation shows that

$$U_{tr} = -\beta cs \int_a^b v''(z(w))\mathrm{d}H_0(w,t) - \beta s(-1)^n \int_a^b v^{(n+1)}(z(w))\left[F_n(w) - G_n(w)\right]\mathrm{d}w,$$

and recalling Equation (4), we find that $(1+r)U_{tr} = sU_{ts}$. As a consequence, $U_{tr}$ and $U_{ts}$ have the same sign[6] and we can directly apply Remark 2 to sign $U_{tr}$: $U_{tr}$ is negative (positive) if $n$th-degree Ross risk aversion is decreasing (increasing) in wealth. Finally, we obtain that

$$\begin{aligned} U_{sr} &= \beta \int_a^b v'(z(w))\mathrm{d}H_1(w,t) + \beta(1+r)s\int_a^b v''(z(w))\mathrm{d}H_1(w,t) \\ &= \beta \int_a^b v'(z(w))\left[1 + (1+r)s\frac{v''(z(w))}{v'(z(w))}\right]\mathrm{d}H_1(w,t), \end{aligned}$$

where the square bracket is non-negative if partial risk aversion is less than unity, which is the case if relative risk aversion is uniformly less than unity (Chiu et al., 2012). Combining the conditions for the various signs accordingly completes the proof.

[6] Under the assumption that individuals save rather than dissave at the optimum, see also Eeckhoudt and Schlesinger (2008).