Public Safety under Imperfect Taxation

Nicolas TREICH

Toulouse School of Economics, INRA, University Toulouse Capitole

Yuting YANG

Toulouse School of Economics, University Toulouse Capitole

[Very Preliminary, please do not circulate]

September 6, 2017

Abstract

In this paper, we examine theoretically the effect of the imperfection of the taxation system on the optimal level of public safety provision. We compare three taxation systems: first-best, income, and uniform taxation. Under wealth heterogeneity, there is normally more public safety under first-best than under uniform taxation, but there can be more or less public safety under first-best than under income taxation. Under risk heterogeneity, the comparison depends on whether the heterogeneity concerns the baseline risk or the reduction in risk. Specifically, under heterogeneous baseline risk, public safety under first-best is lower, and not greater, than under either income or uniform taxation when the utility has constant relative risk aversion. Under heterogeneity in risk reduction, public safety under first-best is in general greater than under uniform or income taxation. We also consider distortionary taxation, namely income taxation under endogenous labor supply. We show that there can be more or less public safety under first-best compared to distortionary taxation. Overall, we conclude that the imperfections of the taxation system cannot generically justify more or less public safety provision. This implies that there is no fundamental reason to always adjust downwards the value of statistical life (VSL) because of imperfect taxation, nor to systematically assume a marginal cost of public funds larger than one for the benefit-cost analysis of public safety projects.

Keywords: Safety provision, imperfect taxation, distortionary taxation, wealth heterogeneity, risk aversion, value of statistical life.

1 Introduction

Mortality reduction represents a significant part of the benefit of many public projects. It has been estimated to account for more than 90% of the monetized benefit of the Clean Air Act (U.S.EPA, 2011). Usually, the standard approach to compute the benefits of safety is to multiply the estimated number of lives saved by the average value of a statistical life (VSL) in the affected population. It is well known however that this common approach in benefit cost analysis relies on the assumption that the taxation system is perfect. In this paper, we relax this assumption, and we examine how the imperfection of the taxation system affects the optimal level of public safety.

The vast majority of the literature on public safety provision has ignored the issue of imperfect taxation. In academia, the literature has examined both theoretically and empirically how VSL varies with the characteristics of individuals or of the decision making environment (Andersson & Treich, 2011; Viscusi & Aldy, 2003). But to the best of our knowledge, this literature has not thoroughly studied the specific effect of imperfect taxation. In the practice of benefit cost analysis, imperfect taxation is often accounted by introducing a marginal cost of public funds larger than one. This practice implies that the cost of the project should be augmented due to imperfect taxation, and in turn that public safety should be reduced. This practice seems problematic because public economics theory is inconclusive in general about whether distributional objectives may lead to a greater or lower level of public expenditures in second best rather than in first best environments (Gaube, 2000).

Accounting for imperfect taxation in the evaluation of public safety projects is important for several reasons. First, it is well documented that the taxation system is imperfect in both developed and developing countries, and that the degree of imperfection vary widely across the world. Second, from a policy perspective, various guidelines encourage safety regulators to also include "distributive impacts," "equity," or "environmental justice" in benefit cost analysis. But it is also well known that concrete methodologies for evaluating such additional impacts remain undeveloped (Adler, 2008). Third, the literature in public economics has long debated the issue of optimal provision of public good when distribution matters, and it seems natural to examine a specific but important domain of application such as safety provision. Indeed, safety usually raises strong equity issues that call for a careful and systematic analysis of distributive incidence. Therefore, it seems important to better understand conceptually the limitation of standard benefit cost analysis under imperfect taxation, and to get more insights about the "biases" induced by this method when the assumption of perfect taxation is relaxed. Accordingly, we believe that a natural starting point in order to get more insights is to develop a comparative statics analysis of the degree of imperfection of the taxation system on the optimal level of public safety.

In our analysis, we proceed as follows. We compare three benchmark types of taxation systems: individual lump-sum taxation (first-best), proportional tax of the uniform rate (income taxation) and uniform taxation. We consider in turn two types of heterogeneity, wealth and mortality risk. Our main results are the following. Under wealth heterogeneity, we show that there is normally more public safety under first-best than under uniform taxation, but that there can be more or less public safety under first-best than under income taxation. Under risk heterogeneity, we show that the comparison depends on whether the heterogeneity concerns the baseline risk or the reduction in risk. Specifically, under heterogeneous baseline risk, public safety is lower than under either income or uniform taxation when the utility has constant relative risk aversion. Under heterogeneity in risk reduction, public safety is in general greater than under uniform or income taxation. We finally consider distortionary taxation, namely the case where income tax rate distort labor supply decisions. We show that there can be more or less public safety under first-best compared to distortionary taxation depending on whether the labor cost is "tangible" (i.e. commensurable with wealth) or not and on weather the labor supply elasticity is positive or negative.

From this analysis, we conclude that the imperfection of the taxation system cannot generically justify more or less public safety provision. In particular, it indicates that there is no a priori reason to adjust downwards the VSL because the expenditures in safety should be financed with imperfect taxation. Similarly, there is no priori reason to systematically assume a marginal cost of public funds larger than one for the benefit cost analysis of safety projects. Instead, these adjustments should be made on a case by case basis and depending on the type of imperfections of the taxation system as well as on the sources of heterogeneity and risk preferences.

Our paper builds on an extensive literature related to the notion of VSL. This notion represents an individual's marginal willingness to pay for a small reduction in mortality risk (Drèze, 1962; Jones-Lee, 1974). Pratt and Zeckhauser (1996) examine the sensitivity of WTP to wealth and baseline mortality risk. Subsequently, researchers have further studied the effect of risk preferences, such as risk aversion (Eeckhoudt & Hammitt, 2004) and background risk (Eeckhoudt & Hammitt, 2001) on VSL. Kaplow (2005) shows that an individual's high coefficient of relative risk aversion could in theory be translated into high income elasticity of VSL, yet the empirical estimates reveal discrepancies. Hammitt and Robinson (2011) discuss the appropriate income elasticity to use when transferring VSL from a high income population to a low income population. One common feature of these studies is that they implicitly assume taxation to be perfect. Armantier and Treich (2004) investigate how heterogeneity on wealth and risk systematically bias the social value of a risk reduction program under uniform taxation. We contribute to the literature by considering the effect of imperfect taxation on VSL.

Our analysis is also related to the literature on the optimal provision of public good under imperfect taxation. The view dates back to Pigou (1947), who states that with distortionary tax, the cost inflicted on consumers exceeds the direct production cost of providing the public good. Thus, at the optimum, the marginal benefit of the public good should be greater than the marginal cost, which results in the under provision of public good. This concept is further formalized into the marginal cost of public fund (MCPF), which was then incorporated into the Samuelson's rule for optimal public good provision.[1] If Pigou's conjecture holds, the value of MCPF should be greater than 1. However, conflicting views have been raised in regards to Pigou's conjecture. Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), Ballard and Fullerton (1992) show that Pigou's conjecture (MCPF>1) hold only under specific circumstances and that MCPF depends on the relationship between the public good, labor supply and other taxed activities. Gaube (2000) states that equity consideration may increase public expenditure in the second best when there are heterogeneous households. This literature has not been applied to public safety provision, which is an important aspect of public good. Thus, we bridge the two literatures by examine the effect of imperfect taxation on optimal safety provision.

[1]Samuelson (1954) provides a basic theory for public good provision: the necessary condition to reach Pareto Optimality of public good provision is to equalize the sum of marginal rates of substitution (MRS) between a public and a private good and the marginal rate of transformation (MRT) $\sum MRS = MRT$. The modified Samuelson's rule has the following form: $\sum MRS = MCPF \times MRT$. See Atkinson and Stern (1974), Ballard and Fullerton (1992).

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 examines the effect of imperfect taxation on public safety under wealth heterogeneity, while section 4 considers mortality risk heterogeneity. Section 5 analyzes the impact of a distortionary tax on labor supply and safety provision. Section 6 discusses the link of our analysis with VSL. Section 7 concludes with policy implications.

2 The Model

2.1 The Framework

We consider a single period model with H individuals. Individual i's expected utility is given by

$$V_i = p_i(G)u(c_i, l_i) + (1 - p_i(G))v(c_i, l_i) \tag{2.1}$$

where $c_i = \omega l_i$. Here $p_i(G)$ denotes the probability of survival given the level of public expenditure on safety G, $u(\cdot)$ is the individual's utility if he survives the period, $v(\cdot)$ is the bequest motive, c_i is his consumption level, l_i is his labor supply and ω is the wage rate. In this paper, we only study the individual heterogeneity in consumption level c_i and risk p_i, thus for simplicity, we assume that all individuals have the same utility function and we normalize bequest motive to zero ($v = 0$).[2]

To carry out the analyses, we first make some assumptions on the form of the utility and survival function. Regarding the utility function, we assume it is strictly positive ($u > 0$), since the bequest motive is normalized to zero, and survival is assumed to be strictly preferred to death.[3] Utility is also assumed to be increasing and concave in the consumption level ($u_c > 0$, $u_{cc} < 0$), but decreasing and concave in labor supply ($u_l < 0$, $u_{ll} < 0$).

For the survival function, we assume it is positive, increasing, and weakly concave ($p_i(\cdot) > 0$, $p_i'(\cdot) > 0$, $p_i''(\cdot) \leq 0$), and $p_i(G) < 1$ for all is. For the simulations, we will use the functional form $p_i(G) = a_i + b_i \frac{G}{1+G}$, where $0 \leq a_i < 1 - b_i$ and $0 \leq b_i \leq 1$.[4]

Suppose a utilitarian social planner is choosing the level of safety provision, e.g. how much to invest in improving air quality. The public good G is financed by individual

[2] The possibility of a bequest motive is discussed in the appendix.
[3] The strong assumption of $u > 0$ is relaxed when we consider the case with bequest in the appendix.
[4] When the specific functions are used, we would verify that the optimal level of public safety $G^* > 0$.

taxation t_i. In order to determine the socially optimal level of public safety, the social planner solves the following welfare maximization problem:

$$\begin{aligned} \max_{G,\{t_i\}_{i\in\{1,\dots H\}}} \quad & \sum_{i=1}^{H} p_i(G)u(c_i, l_i) \\ \text{where} \quad & c_i = \omega l_i - t_i \quad \forall i \\ \text{s.t.} \quad & G \leq \sum_{i=1}^{H} t_i \end{aligned} \tag{2.2}$$

Setting the Lagrangian:

$$\mathcal{L} = \sum_{i=1}^{H} p_i(G)u(\omega l_i - t_i, l_i) + \mu(\sum_{i=1}^{H} t_i - G) \tag{2.3}$$

the first order conditions (focs) with respect to t_i and G give

$$p_i(G^*)u'(\omega l_i - t_i^*, l_i) = \mu, \quad \forall i \tag{2.4}$$

$$\sum_{i=1}^{H} p_i'(G^*)u(\omega l_i - t_i^*, l_i) = \mu \tag{2.5}$$

The focs indicate that a utilitarian social planner would equalize the after tax expected marginal utility of wealth across individuals.

Replacing μ in equation 2.5, we get

$$\sum_{i=1}^{H} p_i'(G^*)VSL_i = 1 \tag{2.6}$$

$$\text{where} \quad VSL_i \equiv \frac{u(\omega l_i - t_i^*, l_i)}{p_i(G^*)u'(\omega l_i - t_i^*, l_i)}$$

where $G^* = \sum_{i=1}^{H} t_i^*$. VSL_i is the value of statistical life (VSL) of each individual i, which describes the marginal rate of substitution (MRS) between wealth and survival probability. VSL exhibits two standard effects, namely the dead-anyway effect and the wealth effect. The dead-anyway effect (Pratt & Zeckhauser, 1996) states that VSL decreases in the survival probability p, i.e. individual facing higher risks has incentive to increase his spending on risk reductions. The wealth effect states that VSL increases in wealth, i.e. the richer the individual, the more he will spend on risk reduction. In this framework, wealth is equivalent to the consumption level each individual.

Equation 2.6 characterizes the efficiency condition to achieve the optimal level of public safety provision. It corresponds to the Samuelson's condition (Samuelson, 1954) of equalizing social marginal benefit to the social marginal cost of providing for the public good (which equals to one here).

To achieve this first-best level of public safety provision, individual lump-sum tax is used. We denote this tax system as "perfect taxation", as there is perfect redistribution without any distortion. However, in practice, perfect taxation is difficult to implement due to informational and political constraints. Therefore, the imperfection in taxation may arise from two sources: 1) The existing tax scheme cannot fully account for the heterogeneous characteristics of individuals and result in imperfect redistribution. 2) The tax system may distort labor supply, thus creating dead-weight loss. In this paper, we study two types of imperfect tax schemes in particular: uniform tax and income tax, which either lacks redistribution or generates distortion. The purpose of this paper is to understand how the underlying imperfections of the taxation systems affect the optimal level of public safety.

2.2 The Benchmark

In section 3 and 4, we focus on individual heterogeneity and assume wealth (w_i) to be exogenous and independent of labor supply. Therefore, we abstract from the general utility form and focus on the utility of consumption $u(c_i)$, where $c_i = w_i - t_i$. To study the effect of heterogeneity, we consider a simple model with two agents who differ either in wealth or survival probability.

The benchmark (first-best) case in section 3 and 4 is the following:

$$\max_{t_1,t_2} p_1(t_1+t_2)u(w_1-t_1) + p_2(t_1+t_2)u(w_2-t_2) \tag{2.7}$$

The focs are

$$\begin{aligned} p_1'(t_1^*+t_2^*)u(w_1-t_1^*) + p_2'(t_1^*+t_2^*)u(w_2-t_2^*) &= p_1(t_1^*+t_2^*)u'(w_1-t_1^*) \\ &= p_2(t_1^*+t_2^*)u'(w_2-t_2^*) \end{aligned} \tag{2.8}$$

As in the general case, the focs imply that the first best optimal tax level would equalize individual's after tax expected marginal utility of wealth. As a result, an individual with higher ex-ante wealth level or lower ex-post probability of survival is subject to a higher tax rate. [5] In these sections, we compare the optimal level of public safety, or equivalently the sum of optimal tax level, under perfect and imperfect taxation. The imperfect tax systems considered are a uniform lump-sum tax for all individuals and an income tax with a uniform tax rate. Wealth and risk heterogeneity are studied separately to understand the effect of each type of individual heterogeneity.

In these sections, the utility function is assumed to be thrice differentiable. To illustrate the analyses, two common utility forms will be used, namely constant relative risk aversion (CRRA) utility and exponential utility. With CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \gamma \in (0,1)$;[6] with exponential utility, $u(c) = \frac{1-e^{-\alpha c}}{\alpha}, \alpha > 0$. Two analytically important coefficients are relative risk aversion $R(c) = -c\frac{u''(c)}{u'(c)}$ and fear of ruin $FR(c) = \frac{u(c)}{u'(c)}$ (Foncel & Treich, 2005). The only class of utility function that has linear fear of ruin is CRRA, which also has the property that $R(c) = \gamma$.

In section 5, we study the effect of distortionary tax on the optimal level of public safety. Here the individual heterogeneities are neglected and we move back to the general utility form $u(c, l)$. The benchmark refers to the case with a non-distortionary lump-sum tax, which raises tax revenue without distorting labor supply decisions. In this section, we compare the level of public safety under the benchmark and income tax.

3 Wealth Heterogeneity

First, we look at the case of wealth heterogeneity among individuals. The first best refers to perfect taxation with individualized tax rate (t_i). Deviating from the first best, we consider two cases: uniform tax (t_U) and income tax (τ). Thus, the utilitarian social planner solves the following maximization problems under the three tax regimes.

First-best:

$$\max_{t_1,t_2} p(t_1 + t_2)[u(w_1 - t_1) + u(w_2 - t_2)] \tag{3.1}$$

[5]Formally, assuming $p_1(\cdot) = p_2(\cdot)$, $w_1 > w_2$, focs 2.8 indicates $u'(w_1 - t_1^*) = u'(w_2 - t_2^*)$. So it must be that $t_1^* > t_2^*$. Assuming $w_1 = w_2 = w$, $p_1(\cdot) > p_2(\cdot)$, focs 2.8 indicates $u'(w - t_1^*) < u'(w - t_2^*)$. So it must be that $w - t_1^* > w - t_2^*$, and $t_1^* < t_2^*$.

[6]A more general form of CRRA utility is $u(c) = \begin{cases} \frac{c^{1-\gamma}-1}{1-\gamma}, \gamma \neq 1 \\ ln(c), \gamma = 1 \end{cases}$. However, in this paper, when CRRA is mentioned, we refer to the specific form of $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Uniform Tax:

$$\max_{t_U} p(2t_U)[u(w_1 - t_U) + u(w_2 - t_U)] \tag{3.2}$$

Income Tax:

$$\max_{\tau} p\big(\tau(w_1 + w_2)\big)[u(w_1(1-\tau)) + u(w_2(1-\tau))] \tag{3.3}$$

Rearranging the focs of the decision variables, we can easily get the following equations:

First-best:

$$\frac{p(G_F)}{p'(G_F)} = \frac{u(w_1 - t_1^*)}{u'(w_1 - t_1^*)} + \frac{u(w_2 - t_2^*)}{u'(w_2 - t_2^*)} \tag{3.4}$$
$$\text{where} \quad w_1 - t_1^* = w_2 - t_2^*$$

Uniform Tax:

$$\frac{p(G_U)}{p'(G_U)} = 2\frac{u(w_1 - t_U^*) + u(w_2 - t_U^*)}{u'(w_1 - t_U^*) + u'(w_2 - t_U^*)} \tag{3.5}$$

Income Tax:

$$\frac{p(G_I)}{p'(G_I)} = (w_1 + w_2)\frac{u(w_1(1-\tau^*)) + u(w_2(1-\tau^*))}{w_1 u'(w_1(1-\tau^*)) + w_2 u'(w_2(1-\tau^*))} \tag{3.6}$$

where $G_F = t_1^* + t_2^*$, $G_U = 2t_U^*$ and $G_I = \tau^*(w_1 + w_2)$.[7] Assuming $w_1 > w_2$, we can infer from equation 3.4 that $t_1^* > t_2^*$.

To compare the optimal level of public safety to provide, we are interested in the ranking of G_F, G_U and G_I. We separately compare first-best with uniform tax and income tax in the remainder of this section.

3.1 First Best and Uniform Tax Comparison

Proposition 1. *Under wealth heterogeneity, with $u'''(x) \geq 0$, the optimal level of public safety in the first-best is higher than that with uniform taxation ($G_F > G_U$).*

Proof. We demonstrate the result by contradiction.

Equation 3.4 can be rewritten as

$$\frac{p(G_F)}{p'(G_F)} = 2\frac{u(w_1 - t_1^*) + u(w_2 - t_2^*)}{u'(w_1 - t_1^*) + u'(w_2 - t_2^*)} \tag{3.7}$$

[7]Given the assumptions made, the second-order conditions hold globally.

Observe that the left-hand side (LHS) of equation 3.5 and 3.7 are of the same form, and is an increasing function of G. Therefore, we only need to compare the right-hand side (RHS) of the two equations to determine the safety level. Assume $\frac{t_1^*+t_2^*}{2} \leq t_U^*$, then

$$\begin{aligned} u(w_1 - t_U^*) + u(w_2 - t_U^*) &\leq u(w_1 - \frac{t_1^* + t_2^*}{2}) + u(w_2 - \frac{t_1^* + t_2^*}{2}) \quad \text{by } u' > 0 \\ &< u(w_1 - t_1^*) + u(w_2 - t_2^*) \quad \text{since } w_1 - t_1^* = w_2 - t_2^* \text{ and } u'' < 0 \end{aligned}$$

Similarly, we have

$$\begin{aligned} u'(w_1 - t_U^*) + u'(w_2 - t_U^*) &\geq u'(w_1 - \frac{t_1^* + t_2^*}{2}) + u'(w_2 - \frac{t_1^* + t_2^*}{2}) \quad \text{by } u'' < 0 \\ &\geq u'(w_1 - t_1^*) + u'(w_2 - t_2^*) \quad \text{since } w_1 - t_1^* = w_2 - t_2^* \text{ and } u''' \geq 0 \end{aligned}$$

Therefore, under $u''' \geq 0$, we have

$$\frac{p(G_U)}{p'(G_U)} = 2\frac{u(w_1 - t_U^*) + u(w_2 - t_U^*)}{u'(w_1 - t_U^*) + u'(w_2 - t_U^*)} < 2\frac{u(w_1 - t_1^*) + u(w_2 - t_2^*)}{u'(w_1 - t_1^*) + u'(w_2 - t_2^*)} = \frac{p(G_F)}{p'(G_F)}$$

Since $\frac{p(G)}{p'(G)}$ is an increasing function in G, $\frac{p(G_U)}{p'(G_U)} < \frac{p(G_F)}{p'(G_F)}$ implies $G_U < G_F$ and $\frac{t_1^*+t_2^*}{2} > t_U^*$, a contradiction. Therefore, it must be $\frac{t_1^*+t_2^*}{2} > t_U^*$. □

Proposition 1 shows that, for utility functions whose third derivative is non-negative, the optimal level of public safety is higher in the first best than in uniform tax. The assumption $u''' \geq 0$, or prudence (Kimball, 1990), is common in the literature. It is a necessary condition for decreasing absolute risk aversion.

The intuition of this result lies in the fact that taxation as a policy instrument serves two purposes: redistribution and financing. In the first best, tax rates are individually tailored, thus both purposes could be fulfilled. In other words, the social planner can choose the optimal level of public safety that maximizes social welfare, and pick different tax rates for each individual that make both individuals equal in their utility level. Under the uniform taxation scheme, however, the social planner's hands are tied to tax individuals at the same level. Therefore, with wealth heterogeneity, the expected marginal utility of consumption of the two individuals cannot be equalized as there is no redistribution in the wealth dimension. With the utility concave in the consumption level, tax of the same amount has a larger impact on the poor than on the rich. Hence, the social planner has to trade off between the first best level of public safety and the welfare of the poor. As a result, a lower safety level is provided.

To illustrate the result with a specific and extreme example, consider two individuals with wealth 1000 and 10 respectively. They both have the same CRRA utility $u(c_i) = \frac{c_i^{0.5}}{0.5}$ and survival function $p(G) = \frac{G}{1+G}$. In the first best, the rich is taxed 516.7 and the poor is given a subsidy of 473.3. Therefore, the total investment on public safety is 43.5. Under uniform taxation, each individual is taxed 7.1 and the total investment on public safety is now 14.2, which is only one third of the level of investment in the first best. With an extremely poor individual, the social planner has to reduce the level of safety investment so that the poor is not too poor in the end. Hence, it is straight forward that the level of public safety under uniform tax must be lower than that in the first best.

3.2 First Best and Income Tax Comparison

Remark 1. *Under wealth heterogeneity, the optimal level of public safety in the first-best could be above, below or equal to the level with income tax.*

Table 3.1 presents the simulations on the optimal public safety provision under three specific cases. We consider CRRA and exponential utility. With CRRA utility, the optimal level is the same under first-best and income tax. With exponential utility, when the degree of risk preference (parameter α in the utility function) varies, the level of provision may be higher or lower in first best than with income tax.

Table 3.1: Simulations of first-best and income tax

Utility		CRRA	Exponential	
Parameter value		$\gamma = 0.5$	$\alpha = 0.02$	$\alpha = 0.001$
Tax Rate	t_1	276.64	580.727	272.87
	t_2	-223.354	80.727	-227.13
	$\tau(w_1 + w_2)$	53.286	493.5	51
Welfare	First Best	105.599	99.826	1101.3
	Income Tax	104.079	99.736	983.15
		$G_F = G_I$	$G_F > G_I$	$G_F < G_I$

Note: Simulated in Mathematica. Taking $p(x) = \frac{x}{1+x}$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, exponential utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $w_1 = 1000$ and $w_2 = 500$.

Table 3.1 indicates that the optimal level of public safety depends on the specific class of utility function and its parameters. In the following, we further study the case of CRRA utility and check whether the level of public safety in the first-best and income tax are always equal.

Remember that CRRA utility has the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. With the assumption of concave non-negative consumption utility, we have $0 < \gamma < 1$. Therefore, $u'(c) = c^{-\gamma}$ and the fear of ruin coefficient $FR(c) = \frac{u(c)}{u'(c)} = \frac{c}{1-\gamma}$ is linear in c.

Substituting the utility function into equation 3.4 and 3.6, we get

First-Best:

$$\frac{p(G_F)}{p'(G_F)} = \frac{w_1 + w_2 - G_F}{1-\gamma} \tag{3.8}$$

Income Tax:

$$\frac{p(G_I)}{p'(G_I)} = \frac{w_1 + w_2 - G_I}{1-\gamma} \tag{3.9}$$

where $G_F = t_1^* + t_2^*$ and $G_I = \tau^*(w_1 + w_2)$. Notice that both equations hold when $\tau^*(w_1 + w_2) = t_1^* + t_2^*$.

Remark 2. *Under wealth heterogeneity, if the utility function satisfies CRRA, then the optimal level of public safety in the first-best is always the same as that with income taxation ($G_F = G_I$).*

Remark 2 suggests that, unlike in the uniform tax case, if the utility satisfies CRRA, the optimal level of safety under income tax is equal to the first best level. The linear fear of ruin property of CRRA utility is instrumental to this result. Notice that with CRRA utility, the RHS of equation 3.9 indicates that only the sum of wealth matters for the social planner when he optimizes the level of public safety. Therefore, if the sum of wealth remains the same, optimal level of public safety would always coincide in the first-best and in income tax, regardless of how the wealth is distributed.

3.3 Wealth Inequality

We have just seen from the above analysis that wealth distribution does not matter for the optimal level of public safety under income taxation if utility satisfies CRRA. In this section, we further analyze the effect of wealth inequality on the level of public safety in each tax scheme.

Assume $w_1 = (1+\eta)\bar{w}$, $w_2 = (1-\eta)\bar{w}$, where $\bar{w}$ denotes the average wealth in the population. Here parameter η measures wealth inequality and $0 \leq \eta < 1$ with $\eta = 0$ indicating perfect equality.

Proposition 2. *An increase in wealth inequality*

(i) does not affect the first-best optimal level of public safety;

(ii) reduces the optimal safety level in uniform tax if $u''' \geq 0$.

Proof. In the first best, the optimal level of taxation is characterized by $w_1 - t_1^* = w_2 - t_2^*$ from equation 3.4. A change in wealth inequality is equivalent as $w_1' = w_1 + \varepsilon$ and $w_2' = w_2 - \varepsilon$. However, the optimality condition must hold and $w_1 + \varepsilon - T_1^* = w_2 - \varepsilon - T_2^*$. It is straight forward that $t_1^* = T_1^* - \varepsilon$ and $t_2^* = T_2^* + \varepsilon$. It follows that $T_1^* + T_2^* = t_1^* + t_2^*$. This proves assertion (i).

Rewrite equation 3.5 as a function of η:

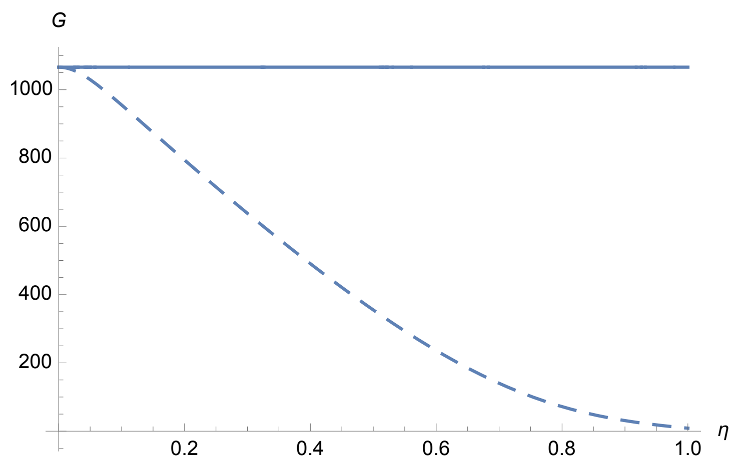
$$F(t_U^*, \eta) = p(2t_U^*)(u_1' + u_2') - 2p'(2t_U^*)(u_1 + u_2) = 0 \tag{3.10}$$

where $u_1 = u((1+\eta)\bar{w} - t_U^*)$ and $u_2 = u((1-\eta)\bar{w} - t_U^*)$. Applying the Implicit Function Theorem, it is easy to obtain that

$$\frac{\mathrm{d}t_U^*}{\mathrm{d}\eta} = -\frac{F_\eta}{F_{t_U^*}} < 0 \tag{3.11}$$

if $u''' \geq 0$. Therefore, assuming prudence, t_U^* decreases in η. This proves assertion (ii). $\square$

Figure 3.1: Effect of wealth inequality on the optimal safety level



Note: Solid line indicates the optimal safety level in the first best. Dashed line indicates the optimal safety level with uniform tax. Simulated in Mathematica. Taking $p(x) = \frac{x}{1+x}$, exponential utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$ and $w = 1000$.

Figure 3.1 illustrates proposition 2. Proposition 2(i) follows the well known result of private provision of public good: under identical preferences, wealth redistribution among contributors will not change the equilibrium supply of public good (Bergstrom, Blume, & Varian, 1986).

Remark 3. *An increase in wealth inequality*

(i) does not affect the income tax optimal level of public safety if utility satisfies CRRA;

(ii) may increase, decrease or maintain the optimal public safety level in income tax with other utility forms.

Proof. Rewrite equation 3.6 as a function of η:

$$F(\tau^*, \eta) = p_{\tau^*}\bar{w}\Big((1+\eta)u_1' + (1-\eta)u_2'\Big) - 2\bar{w}p_{\tau^*}'(u_1 + u_2) = 0 \tag{3.12}$$

where $p_{\tau^*} = p(2\tau^*\bar{w})$, $u_1 = u((1+\eta)\bar{w}(1-\tau^*))$ and $u_2 = u((1-\eta)\bar{w}(1-\tau^*))$. Applying the Implicit Function Theorem, we have

$$\frac{\mathrm{d}\tau^*}{\mathrm{d}\eta} = -\frac{F_\eta}{F_{\tau^*}} \tag{3.13}$$

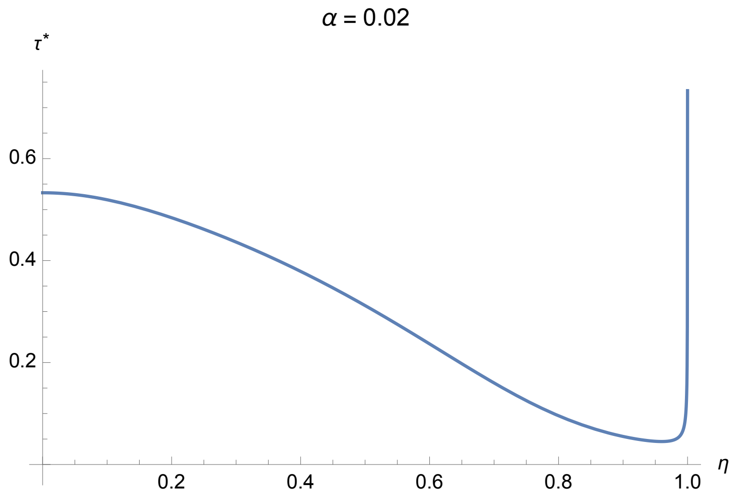
where $F_{\tau^*} < 0$ and $F_\eta = (u_1' - u_2')(2p_{\tau^*}'(1-\tau^*)\bar{w} - p_{\tau^*}) + p_{\tau^*}(1-\tau^*)\bar{w}\Big((1-\eta)u_2'' - (1+\eta)u_1''\Big)$ When utility satisfies CRRA, plug in equation 3.6 into F_η, we get $F_\eta = 0$. Thus, $\frac{d\tau^*}{d\eta} = 0$, the optimal level of public safety is independent of the degree of wealth inequality. This proves assertion (i).

With other utility forms, we cannot conclude the sign of F_η. Figure 3.2 shows an example with exponential utility, taking the parameter value α at 0.02. We can easily see that with different degree of wealth inequality, the optimal income tax rate may increase or decrease given different parameter values of the utility function. This justifies assertion (ii). □

3.4 Welfare Analysis

We have observed that for CRRA utility, public safety level under income tax and first best are the same. Does that imply using income tax is as good as using individual lump-sum taxes? Welfare analysis has to be carried out to reach a conclusion.

Figure 3.2: Effect of wealth inequality on optimal income tax rate



Note: Simulated in Mathematica. Taking $p(x) = \frac{x}{1+x}$, exponential utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$ and $w = 1000$.

Remark 4. *Under wealth heterogeneity, social welfare at the optimum level under income tax is always higher than that under uniform tax, but lower than the first best.*

Proof. We only need to show welfare comparison at the optimal level of public safety under different tax regimes.

Between first best and income tax, it is always possible to choose t_1, t_2 such that $t_1 = \tau^* w_1$ and $t_2 = \tau^* w_2$, so that the welfare under individual taxation can do at least as well as under income tax. However, the first best can never obtain with such t_1 and t_2. As stated in equation 3.4, the optimality condition gives $w_1 - t_1^* = w_2 - t_2^*$. Setting $t_i = \tau^* w_i$ can not satisfy this condition. Therefore, $t_i^* \neq \tau^* w_i$ holds in general and the welfare level in the first best is strictly higher than that in income tax.

Between income and uniform tax, we have for every given level of G such that $G = \tau(w_1 + w_2) = 2t_U$,

$$p(G)(u(w_1 - \tau w_1) + u(w_2 - \tau w_2)) > p(G)(u(w_1 - t_U) + u(w_2 - t_U))$$

by the concavity of the utility function. Therefore, for t_U^*, there exists a τ such that $\tau = \frac{2t_U^*}{w_1+w_2}$, that makes the welfare of income tax higher than uniform tax. Thus, the welfare with income tax at the optimum is always greater than that with uniform tax. □

4 Mortality Risk Heterogeneity

In this section, we consider individual heterogeneity on mortality risk $p_i(G)$ assuming $p_1(G) > p_2(G)$. The heterogeneity may come from two sources: baseline risk and risk reduction. For example, individuals may be exposed to different degrees of risk prior to the implementation of a mortality risk reduction project, or that some may benefit more from the project while others less. As shown in the benchmark case in section 2.2, perfect taxation would impose a higher tax on the agent that has a lower survival probability p_i due to the dead-anyway effect (Pratt & Zeckhauser, 1996). However, with imperfect taxation, the lack of individual tax rate may lead to different levels of optimal public safety than in the first-best.

The social planner's problems can be written as follows.

First-Best:

$$\max_{t_1,t_2} p_1(t_1+t_2)u(w-t_1) + p_2(t_1+t_2)u(w-t_2) \tag{4.1}$$

Uniform Tax:

$$\max_{t_U} \Big(p_1(2t_U) + p_2(2t_U)\Big)u(w-t_U) \tag{4.2}$$

Income Tax:

$$\max_{\tau} \Big(p_1(2\tau w) + p_2(2\tau w)\Big)u(w(1-\tau)) \tag{4.3}$$

Note that income tax is equivalent to uniform tax in this scenario as there is no heterogeneity in wealth. Indeed, one can always set $\tau = \frac{t_U}{w}$ to have $w(1-\tau) = w - t_U$ and obtain $G_I = G_U$. Therefore, we just focus our analysis on the uniform tax case.

Rearranging the focs we get:

First-Best:

$$\begin{aligned}\frac{p_1(G_F)+p_2(G_F)}{p_1'(G_F)+p_2'(G_F)} =& \underbrace{\frac{u(w-t_1^*)}{u'(w-t_1^*)} + \frac{u(w-t_2^*)}{u'(w-t_2^*)}}_{U_{FB}} \\ &+ \underbrace{\frac{p_2(G_F)}{p_1'(G_F)+p_2'(G_F)}}_{A} \underbrace{\frac{u(w-t_1^*)-u(w-t_2^*)}{u'(w-t_1^*)}}_{B} \underbrace{\left(\frac{p_1'(G_F)}{p_1(G_F)} - \frac{p_2'(G_F)}{p_2(G_F)}\right)}_{C}\end{aligned} \tag{4.4}$$

Uniform Tax:

$$\frac{p_1(G_U)+p_2(G_U)}{p_1'(G_U)+p_2'(G_U)} = \underbrace{\frac{2u(w-t_U^*)}{u'(w-t_U^*)}}_{U_{Uni}} \tag{4.5}$$

where $G_F = t_1^* + t_2^*$, $t_1^* < t_2^*$ and $G_U = 2t_U^*$.[8] Again, we are interested in the comparison between G_F and G_U.

We can compare G_F and G_U by looking at the RHS of equation 4.4 and 4.5. First we examine U_{FB} of equation 4.4 and U_{Uni} of equation 4.5. When $\frac{u}{u'}$ is weakly convex, e.g. with CRRA, linear or exponential utility, we have $U_{FB} \geq U_{Uni}$. Given the assumptions we made on the functional forms, we know that $A > 0$ and $B > 0$. Hence if we can pin down the sign of C, we would have a clear idea of the ranking of G_F and G_U. Under CRRA and linear utility, if $\frac{p_1'(G_F)}{p_1(G_F)} \geq \frac{p_2'(G_F)}{p_2(G_F)}$, it follows that $C \geq 0$, and the first-best optimal level must be equal to or higher than with uniform tax ($G_F \geq G_U$). If $\frac{p_1'(G_F)}{p_1(G_F)} < \frac{p_2'(G_F)}{p_2(G_F)}$, $C < 0$, then first-best optimal level would be lower than with uniform tax ($G_F < G_U$). Under exponential utility, if $C \geq 0$, then $G_F > G_U$; if $C < 0$, the result is uncertain.

As mentioned in the example above, heterogeneity in risk may be conceived in two distinct ways: baseline risk and risk reduction. In the remainder of this section, we separately analyze heterogeneous baseline risk and heterogeneous risk reduction to obtain further understanding.

4.1 Heterogeneous Baseline Risk and Risk Reduction

With heterogeneous baseline risk, agents have different baseline survival probability p_i, but receive the same level of benefit from the public safety project $\varepsilon(G)$. The survival function could be expressed as $p_i(G) = p_i + \varepsilon(G)$. Here we make the assumptions that $\varepsilon(\cdot) < 1 - \min\{p_1, p_2\}$, and is positive, increasing and weakly concave ($\varepsilon(\cdot) > 0$, $\varepsilon'(\cdot) > 0$, $\varepsilon''(\cdot) \leq 0$). Assuming $p_1 > p_2$, then

$$\frac{p_1'(G_F)}{p_1(G_F)} = \frac{\varepsilon'(G_F)}{p_1+\varepsilon(G_F)} < \frac{\varepsilon'(G_F)}{p_2+\varepsilon(G_F)} = \frac{p_2'(G_F)}{p_2(G_F)}$$

Thus $C < 0$.

Remark 5. *Under heterogeneous baseline risk,*

[8] For the second order conditions, see Appendix A.2.

1. *if utility is linear or CRRA, optimal level of public safety is lower in the first-best than with uniform or income tax ($G_F < G_U = G_I$);*

2. *if utility if exponential, the relationship between the optimal level of public safety in the first-best and uniform tax or income tax is uncertain.*

Table 4.1: Simulation for heterogeneous baseline risk

Utility		CRRA	Exponential
Parameter Value		$\gamma = 0.5$	$a = 0.02$
Tax Rate	t_1	-71.205	667.821
	t_2	342.999	679.207
	t	136.936	673.232
Welfare	First Best	97.402	88.083
	Income Tax	96.690	88.082
		$G_F < G_U$	$G_F > G_U$

Note: Simulated in Mathematica. Taking $p(G) = p_i + \frac{mG}{1+2mG}$, $p_1 = 0.5$, $p_2 = 0.3$, $m = 0.01$. CRRA utility $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$. Exponential utility $U(w) = \frac{1-e^{-\alpha w}}{\alpha}$, with $\alpha = 0.02$, initial wealth $w = 1000$.

Table 4.1 shows that depending on the utility form, optimal provision of safety could be lower or higher with uniform tax than in the first best.

With heterogeneous risk reduction, agents have the same baseline survival probability p, but benefit from the safety project to different degrees δ_i. The survival function is linear in the public safety level, then $p_i(G) = p + \delta_i G$, $\delta_i < \frac{1-p}{G}$ for any G, and $\delta_1 > \delta_2$. Therefore,

$$\frac{p_1'(G_F)}{p_1(G_F)} = \frac{\delta_1}{p + \delta_1 G_F} > \frac{\delta_2}{p + \delta_2 G_F} = \frac{p_2'(G_F)}{p_2(G_F)}$$

Thus $C > 0$, and $G_F > G_U$ for all utility functions that have $\frac{u}{u'}$ weakly convex.

Remark 6. *Under heterogeneous linear risk reduction ($p_i(G) = p + \delta_i G$), the optimal level of safety provision in the first-best is higher than that with uniform or income tax ($G_F > G_U = G_I$) for all utility functions that have $\frac{u}{u'}$ weakly convex .*

Perfect taxation imposes a lower tax rate on the agent that is more responsive to the safety project to achieve both efficiency and equity ($t_1^* < t_2^*$). Under imperfect taxation, providing lower safety could reduce the gap between the survival probabilities of individuals. Therefore, it is optimal for the social planner to set G_U to be lower than G_F.

4.2 Risk Inequality

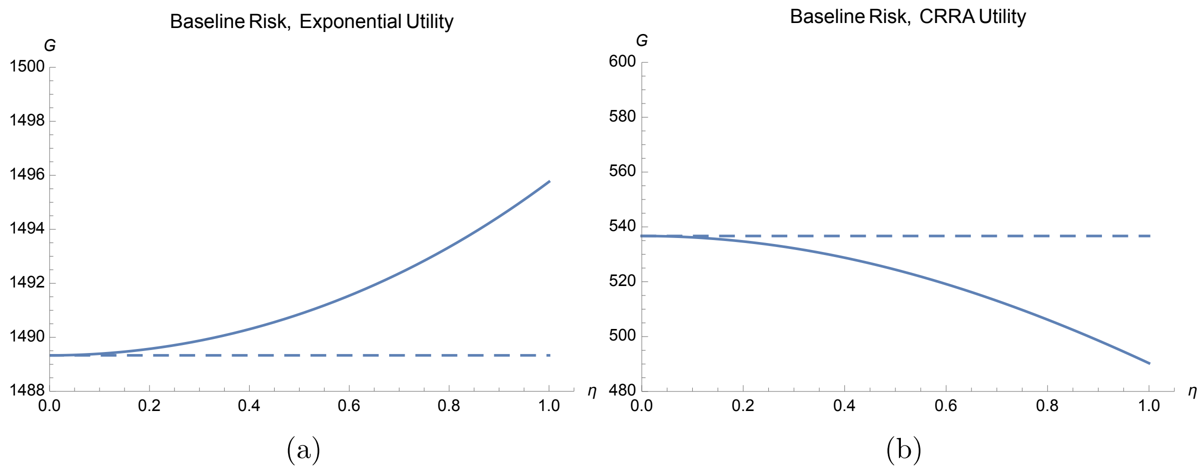
Similar to wealth inequality, risk inequality may also affect the optimal level of public safety under perfect and imperfect taxation. In this part, we analyze the effect of baseline risk inequality and risk reduction inequality.

Proposition 3. *An increase in risk inequality (both baseline risk and risk reduction) does not affect the optimal level of public safety under uniform tax.*

Proof. For baseline risk inequality, we set $p_1(G) = (1+\eta)\bar{p} + \varepsilon(G)$ and $p_2(G) = (1-\eta)\bar{p} + \varepsilon(G)$. η denotes the degree of inequality of baseline risk and $0 \leq \eta < 1$. As the foc of uniform tax is independent from η, G_U remains constant as η increases.

For risk reduction inequality, we set $p_1(G) = p + (1+\eta)\bar{\delta}G$ and $p_2(G) = p + (1-\eta)\bar{\delta}G$. Replacing p_1 and p_2 in equation 4.5, it is easy to see that η no longer plays a role. □

Figure 4.1: Effect of baseline risk inequality on the optimal safety level

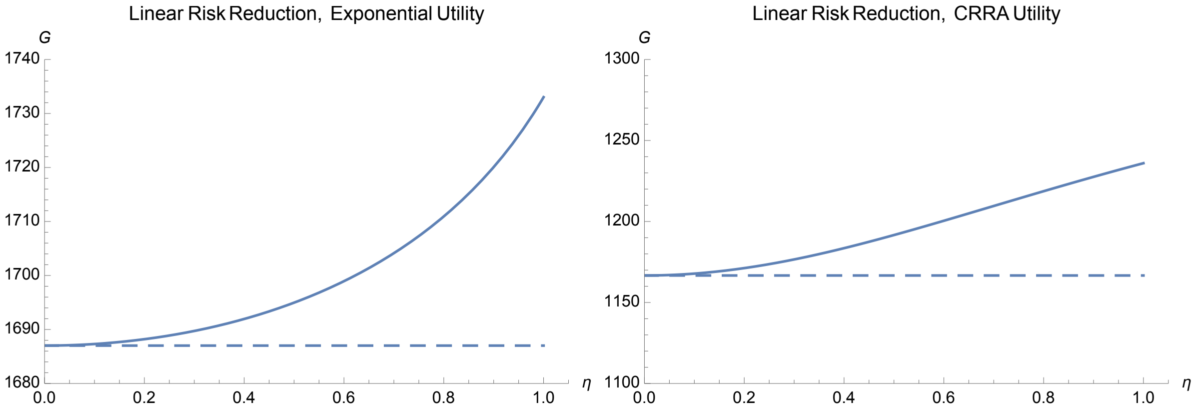


Note: Solid line indicates the optimal safety level in the first best. Dashed line indicates the optimal safety level with uniform tax. Simulated in Mathematica. Taking $p_i(x) = p_i + \frac{0.02x}{1+0.04x}$, $p_1 = (1+\eta)\bar{p}$, $p_2 = (1-\eta)\bar{p}$, $\bar{p} = 0.25$, exponential utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $\gamma = 0.5$ and $w = 1000$.

Changing the distribution of risk affects the optimal level of safety in the first best. However, the direction of change depends on the type of utility function. Figure 4.1 shows the effect of baseline risk inequality on the optimal safety level under exponential and CRRA utility. Under exponential utility, optimal safety level in the first best increases as there is more inequality. However, under CRRA utility, the level decreases.

Similar results hold for the case of risk reduction inequality. As is shown in figure 4.2a, under exponential utility, optimal safety level in the first best increases as there is

Figure 4.2: Effect of risk reduction inequality on the optimal safety level



(a) (b)

Note: Solid line indicates the optimal safety level in the first best. Dashed line indicates the optimal safety level with uniform tax. Simulated in Mathematica. Taking $p_i(x) = p + \delta_i x$, $\delta_1 = (1+\eta)0.0002$, $\delta_2 = (1-\eta)0.0002$, $p = 0.1$, exponential utility $u(x) = \frac{1-e^{-\alpha x}}{\alpha}$, $\alpha = 0.02$, CRRA utility $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, $\gamma = 0.5$ and $w = 1000$.

more inequality. Yet the affect of inequality under CRRA utility reversed, i.e. with more inequality in risk reduction, the optimal level of public safety in the first best increases.

5 Distortionary Taxation

In this section, we focus on the distortionary aspect of imperfect taxation. Therefore, we abstract from heterogeneous individuals and look at identical agents with labor supply. Each individual maximizes his utility by choosing the level of consumption (c) and labor supply (l), subject to the tax schedule (t) that he faces. Individuals are assumed to be small so they do not take into account the feedback effect of taxation.[9] The social planner's objective is then to set the optimal tax rate (t^*) that maximizes social welfare, while fully anticipating the response of agents to the tax rate chosen. Following standard public economics literature, the first-best refers to lump-sum taxation. Here, we take income tax as the imperfect taxation for comparison.

In the first stage, each agent takes the tax rate as given and maximizes utility subject to his budget constraint. Therefore, the individual's problems in the first best and under income tax are:

[9]Individuals take the level of public safety as given. That is, they do not take into consideration that their labor supply level could influence the total amount of safety.

First best:

$$\begin{aligned} \max_{c_t, l_t} \quad & p(G_F)u(c_t, l_t) \\ \text{s.t.} \quad & c_t = wl_t - t \end{aligned} \tag{5.1}$$

Income tax:

$$\begin{aligned} \max_{c_\tau, l_\tau} \quad & p(G_I)u(c_\tau, l_\tau) \\ \text{s.t.} \quad & c_\tau = wl_\tau(1-\tau) \end{aligned} \tag{5.2}$$

The optimal decision in the first best is characterized by $c_t^*(t)$, $l_t^*(t)$: $-\frac{u_l^*}{u_c^*} = w$, where $u_c^* = \frac{\partial u(c_t^*, l_t^*)}{\partial c_t}$ and $u_l^* = \frac{\partial u(c_t^*, l_t^*)}{\partial l_t}$. Under income tax, the optimal decision is $c_\tau^*(\tau), l_\tau^*(\tau)$ such that $-\frac{u_l^*}{u_c^*} = w(1-\tau)$.

In the second stage, the social planner then determines the optimal level of taxation subject to the revenue requirement for public safety provision. The social planner's problems are respectively:

First best:

$$\begin{aligned} \max_{t} \quad & Hp(G_F)u\Bigg(c_t^*(t), l_t^*(t)\Bigg) \\ \text{s.t.} \quad & G_F = Ht \end{aligned} \tag{5.3}$$

Income tax:

$$\begin{aligned} \max_{\tau} \quad & Hp(G_I)u\Bigg(c_\tau^*(\tau), l_\tau^*(\tau)\Bigg) \\ \text{s.t.} \quad & G_I = Hwl_\tau^*(\tau)\tau \end{aligned} \tag{5.4}$$

Rearranging the focs from the two planner's problems, we get

First best:

$$\frac{p(G_F)}{Hp'(G_F)} = \frac{u\Bigg(c_t^*(t), l_t^*(t)\Bigg)}{u_c\Bigg(c_t^*(t), l_t^*(t)\Bigg)} \tag{5.5}$$

Income tax:

$$\frac{p(G_I)}{Hp'(G_I)} = \frac{u\Big(c_\tau^*(\tau), l_\tau^*(\tau)\Big)}{u_c\Big(c_\tau^*(\tau), l_\tau^*(\tau)\Big)}(1 + \varepsilon_{l\tau^*}) \tag{5.6}$$

where $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau} / \frac{l}{\tau}$ denotes the labor supply elasticity of income tax.[10]

Comparing equation 5.5 and 5.6, the only additional term is $1 + \varepsilon_{l\tau^*}$, which could be interpreted as the distortionary effect of income tax, or the marginal cost of public fund (MCPF) as is commonly used in the public economics literature (Browning, 1976). Depending on the value of this labor supply elasticity, the first best public safety provision (G_F/G_U) could be greater or lower than that with a income tax (G_I).

The value of $\varepsilon_{l\tau}$ depends on the properties of the utility function. Here we consider two cases: tangible labor effort (i.e. commensurable with wealth) $u(c, l) = u(c - e(l))$, and non-tangible labor effort $u(c, l) = u(c) - e(l)$ (with $e(l) > 0$, $e'(l) > 0$, $e''(l) > 0$).

Proposition 4. *Under distortionary tax with identical agents,*

1. *if labor effort is tangible, then labor supply decreases with increasing tax rate ($\varepsilon_{l\tau^*} < 0$) and the optimal level of public safety is lower under income tax tax than in the first-best ($G_F = G_U > G_I$).*

2. *if labor effort is non-tangible, the elasticity of labor supply is positive (negative) ($\varepsilon_{l\tau^*} > 0 (< 0)$) if the level of relative risk aversion is greater (lower) than 1 ($R > 1 (< 1)$), and the optimal level of public safety is lower (higher) under income tax than in the first-best ($G_F = G_U > (<) G_I$).*

Proof. As $\varepsilon_{l\tau^*} = \frac{\partial l}{\partial \tau} / \frac{l}{\tau}$ and $\frac{l_t^*(\tau^*)}{\tau^*} > 0$, to obtain the sign of $\varepsilon_{l\tau^*}$, we just need to sign $\frac{\partial l^*}{\partial \tau}$.

1. Tangible labor cost $u(c, l) = u(c - e(l))$, $c = wl(1 - \tau)$

$$\frac{\partial l^*(\tau)}{\partial \tau} = -\frac{\frac{\partial^2 u}{\partial l \partial \tau}}{\frac{\partial^2 u}{\partial^2 l}} = -\frac{w}{e''(l^*)} \tag{5.7}$$

By assumption, $e''(l) > 0$, then $\frac{\partial l^*}{\partial \tau} < 0 \implies \varepsilon_{l\tau^*} < 0 \implies G_F > G_I$.

2. Non-tangible labor cost $u(c, l) = u(c) - e(l)$, $c = wl(1 - \tau)$

$$\frac{\partial l^*(\tau)}{\partial \tau} = \frac{u''(c)w^2 l^*(1 - \tau) + u'(c)w}{u''(c)wl^*(1 - \tau)^2 - e''(l^*)} \tag{5.8}$$

[10] Second order conditions for both the individual problem and planner's problem are satisfied globally.

By assumption, the denominator is negative. If we denote the coefficient of relative risk aversion as $R(c) = -c\frac{u''(c)}{u'(c)}$, in this case $R = -wl(1-\tau)\frac{u''(c)}{u'(c)}$. If $R < 1$, then the numerator of the RHS of equation 5.8 is positive, which implies $\varepsilon_{l\tau^*} < 0$, and it follows that $G_F > G_I$. If $R \geq 1$, then $\varepsilon_{l\tau^*} \geq 0$ and $G_F \leq G_I$.

□

If the utility function satisfies the CRRA form, $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$, then $R(w) = \gamma$. As we have restricted utility to be positive, thus $\gamma \leq 1$. Therefore, with a non-tangible cost of labor, under CRRA and positive utility assumption, we still have $G_F > G_I$. However, with other utility functions, it may occur that $G_F \leq G_I$. For example, for exponential utility, $u(w) = \frac{1-e^{-\alpha w}}{\alpha}$, we have $R = \alpha w$, which could be greater than one if $\alpha > \frac{1}{w}$.

How can we explain this ambiguous result under non-tangible cost of effort? It follows that taxation creates both a substitution effect and an income effect which works in opposite directions in determining optimal labor supply. If tax rate increases, implicitly the consumption good is more expensive compared with leisure. Thus the substitution effect would reduce labor supply or increase the consumption of leisure in order to achieve a higher utility level. However, with higher tax rate, wealth decreases and the marginal utility of wealth increases. On the other hand, the income effect would increase labor supply to avoid being poor. When the substitution effect dominates the income effect, the rise in tax rate would reduce overall labor supply and vice-versa. Essentially, with $R > 1$, the curvature of the utility function is large. Thus, a small decrease in wealth would imply a large increase in the marginal utility. This would result in a larger income effect than the substitution effect, which induce higher labor supply with higher tax rate. When labor effort is "tangible", the curvature of the utility function does not play a role in optimal decision making ($e'(l^*) = w(1-\tau^*)$). Whereas when labor effort is separable from the utility of wealth, the curvature of utility function effectively decides labor supply ($e'(l^*) = w(1-\tau^*)u'(w(1-\tau^*)l^*)$. Thus, only in the "non-tangible" case, the ambiguous result can occur.

Although there is the theoretical possibility that the income effect may dominate the substitution effect and result in increase in labor supply with higher tax rates, there is little empirical evidence of such occurrence (Meghir & Phillips, 2010). However, Manski (2014) argues that the consensuses in the empirical literature may be an artifact of the strong assumptions made in the models.[11] He states that without the knowledge of income-leisure preference, one cannot predict how labor effort may change with tax rate. Our analysis is in line with the conclusion Manski (2014) made.

6 Link with VSL

In practice, policy making agencies that implement safety projects, e.g. the U.S. Environmental Protection Agency (EPA) and the U.S. Department of Transportation (DOT), commonly use the Value of a Statistical Life (VSL) to monetize the value of mortality risk reduction benefit. As mortality risk reduction accounts for the biggest portion of safety benefit, choosing the right VSL becomes vital. Currently, the above mentioned agencies use a single VSL obtained from meta-analysis for all impact evaluations.[12] This VSL is equivalent to the arithmetic mean of individual VSLs (U.S.EPA, 2016).

Although using the average VSL is a common practice in project evaluations, it is at best a shortcut and lacks consideration for individual heterogeneities. Armantier and Treich (2004) also share the concern for assuming away individual heterogeneities in evaluating mortality risk reduction projects. They show that heterogeneity on wealth and baseline risk (respectively on risk reduction) leads to systematical overestimation (respectively underestimation) of the social value of a risk reduction program. Adler (2016) conducts some numerical exercises to calculate the distributional weights for VSLs in the presence of heterogeneities. He shows that under different parameter values, the resulting distributional weights may lead to opposing policy outcomes. To build on this literature, we attempt to provide some insights on the effects of imperfect taxation on the VSL.

Remember from equation 2.6, the foc of the optimization problem can be written as:

$$\sum_{i=1}^{H} p_i'(G^*)VSL_i = 1 \tag{6.1}$$

[11] Manski (2014) wrote, "Examining the models of labor supply used in empirical research, I have become concerned that the prevailing consensus on the sign of uncompensated elasticities may be an artifact of model specification rather than an expression of reality". He pointed out that the two assumptions generally made in the empirical models, non-backward-bending labor supply functions and homogeneous response of labor supply to net wage across populations, may lead to the positive labor supply elasticity of tax.

[12] The existing EPA guidelines suggest using a VSL of $9.7 million in 2013 U.S dollars (U.S.EPA, 2010). According to a document issued by DOT, by 2015, DOT uses a VSL of $9.5 million for their analyses. https://cms.dot.gov/sites/dot.gov/files/docs/VSL2015_0.pdf

where $VSL_i = \frac{u(w_i - t_i^*)}{p_i(G^*)u'(w_i - t_i^*)}$ denotes the individual's marginal rate of substitution between wealth and mortality risk. Observe that only if $p_i'(G)$ is independent from VSL_i, then equation 6.1 is equivalent to

$$\frac{1}{n}\sum_{i=1}^{H} VSL_i = \frac{1}{\sum_{i=1}^{H} p_i'(G^*)} \tag{6.2}$$

which equates the population average VSL to the social marginal cost of saving a life. However, in our framework, $p_i'(G)$ is only independent of VSL_i when there is not heterogeneity in risk, which makes the use of average VSL unjustifiable under risk heterogeneity. Moreover, equation 6.2 also indicates that, in addition to having independent $p_i'(G)$, average VSL is the proper measure of benefit when taxation is "perfect", in the sense that there is perfect redistribution through taxation. Thus, once we divert from perfect taxation, average VSL can no longer be an accurate measure of the optimal public safety level.

From the above analysis, we can infer that the average VSL may create an over- or underestimation of project benefit under imperfect taxation. In the remainder of this section, we study the the direction of bias if average VSL is continually used, under different heterogeneities and tax systems. The results are obtained by comparing average VSL at the optimal tax rate (t^*) with the weighted VSL given by the focs.

6.1 Wealth Heterogeneity

Under wealth heterogeneity, as $p'(G)$ is independent of VSL_i and in the first best perfect taxation is guaranteed, it is straight forward that using average VSL is reasonable.

Under uniform taxation, observing the first order conditions, we could express equation 3.5 as

$$\sum_{i=1}^{2} \lambda_i VSL_i = \frac{1}{2p'(G_U)} \tag{6.3}$$

where $VSL_i = \frac{u(w_i - t_U^*)}{p(G_U)u'(w_i - t_U^*)}$, $\lambda_i = \frac{u'(w_i - t_U^*)}{\sum_j u'(w_j - t_U^*)}$ and $\sum_{i=1}^{2} \lambda_i = 1$.

With $w_1 \neq w_2$, $\lambda_i \neq \frac{1}{2}$, the average VSL must be different from the weighted VSL. The optimality condition implies that individual VSLs should be given different weights according to the marginal utility of wealth. Given the assumptions we made, $u_1' < u_2'$, thus $\lambda_1 < \lambda_2$, which means that the VSL for the poor should be given a higher weight.

This in consistent with the numerical exercises done in Adler (2016). As the VSL of the rich is larger than the poor, this weighted VSL would be lower than the average VSL. Therefore, using the average VSL would result in an overestimation of the project value.

Similarly, in the case of income taxation, equation 3.6 can also be expressed in terms of VSL:

$$\frac{1}{2}\sum_{i=1}^{2}\mu_i VSL_i = \frac{1}{2p'(G_I)}, \tag{6.4}$$

where $VSL_i = \frac{u(w_i(1-\tau^*))}{p(G_I)u'(w_i(1-\tau^*))}$, $\mu_i = \frac{\sum_j w_j u_i'(w_i(1-\tau^*))}{\sum_i w_i u_i'(w_i(1-\tau^*))}$, $\mu_1 + \mu_2 > 2$. With similar reasoning, $\mu_1 < \mu_2$, the poor's VSL is again has more weight than the rich. However, as $\frac{1}{2}(\mu_1 + \mu_2) > 1$, it is uncertain whether taking the average VSL in income tax would over- or underestimate the project benefit. For example, in the case of CRRA utility, $\frac{1}{2}\sum_{i=1}^{2}\mu_i VSL_i = \frac{1}{2}\sum_{i=1}^{2} VSL_i$, where taking the average VSL is equivalent to weighted VSL.

6.2 Risk Heterogeneity

In the case of risk heterogeneity, the marginal mortality risk reduction $p_i'(G)$ may no longer be identical across agents. We can further separate the two case: 1) heterogeneous baseline risk, $p_i'(G) = p'(G)\ \forall i$s; and 2) heterogeneous risk reduction, $p_i'(G) \neq p_j'(G)$.

Observe that with heterogeneous risk reduction, average VSL is not supported even in the first best. The focs of equation 4.1 can be written as:

$$\sum_{i=1}^{2}\theta_i VSL_i = \frac{1}{\sum_{i=1}^{2} p_i'(G_F)} \tag{6.5}$$

where $VSL_i = \frac{u(w-t_i^*)}{p_i(G_F)u'(w-t_i^*)}$, $\theta_i = \frac{p_i'(G_F)}{\sum_j p_j'(G_F)}$ and $\sum_{i=1}^{2}\theta_i = 1$. We have concluded in section 4 that assuming $p_1 > p_2$, at the first best optimum $t_1 < t_2$, thus $VSL_1 < VSL_2$. But given the assumptions, here $\theta_1 > \theta_2$, indicating the VSL of the lucky individual who benefit more from the project is given a higher weight. Hence, using average VSL in the first best would lead to an overestimation of the project value with heterogeneity in risk reduction.

Under uniform tax, we could rewrite equation 4.5 as the following:

$$\sum_{i=1}^{2}\phi_i VSL_i = \frac{1}{\sum_{i=1}^{2} p_i'(G_U)} \tag{6.6}$$

where $VSL_i = \frac{u(w-t_U^*)}{p_i(G_U)u'(w-t_U^*)}$, $\phi_i = \frac{p_i(G_U)}{\sum_j p_j(G_U)}$ and $\sum_{i=1}^{2} \phi_i = 1$. As $p_1 > p_2$, we have $\phi_1 > \phi_2$. In other words, VSL of the lucky individual is again given a higher weight. Similarly, using average VSL under uniform taxation would result in an overestimation of the project benefit with risk heterogeneity.

Although in both tax systems, an overestimation would incur with the use of the average VSL, the extent of the overestimation, however, differs. In other words, the underlying tax systems modifies the weights for different social groups.

Table 6.1: Average VSL and Weighted VSL

	Wealth Heterogeneity		**Risk Heterogeneity**		
			Baseline Risk	Risk Reduction	
	Uniform	Income	Uniform	First best	Uniform
	(1)	(2)	(3)	(4)	(5)
VSL$_1$	\$ 2,020,200	\$ 1,987,530	\$ 1,932,860	\$ 667,393	\$ 1,689,120
VSL$_2$	\$ 404,040	\$ 397,505	\$ 2,014,000	\$ 665,214	\$ 1,797,420
Weights	$\lambda_1 = 0.178$ $\lambda_2 = 0.822$	$\mu_1 = 0.624$ $\mu_2 = 2.879$	$\phi_1 = 0.510$ $\phi_2 = 0.490$	$\theta_1 = 0.667$ $\theta_2 = 0.333$	$\phi_1 = 0.516$ $\phi_2 = 0.484$
Average VSL	\$ 1,212,121	\$ 1,192,520	\$ 1,973,430	\$ 666,303	\$ 1,743,270
Weighted VSL	\$ 691,952	\$ 1,192,520	\$ 1,972,600	\$ 666,667	\$ 1,741,590
$\frac{\textbf{Average VSL}}{\textbf{Weighted VSL}}$	1.752	1	1.00042	1.00042	1.00097

Note: Simulated in Mathematica. VSL$_i$ denotes the VSL of the corresponding individual at optimal tax rate. Weights for the individuals follow the notation in the corresponding parts in this section. Average VSL is calculated as $\frac{VSL_1+VSL_2}{2}$. Weighted VSL is calculated as, for example in Column (1), $\lambda_1 VSL_1 + \lambda_2 VSL_2$. Assume all individuals have the same CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma = 0.95$. Column (1) and (2) are cases of wealth heterogeneity, with $w_1 = \$100,000$, $w_2 = \$20,000$ and $p(x) = 0.99 + 0.01\frac{x}{20000+x}$. Column (3) is the case of baseline risk heterogeneity, with $w = \$100,000$, $p_1(x) = 0.99 + 0.01\frac{x}{20000+x}$, $p_2(x) = 0.95 + 0.01\frac{x}{20000+x}$. Columns (4) and (5) are cases of linear risk reduction heterogeneity, with $p_1(x) = 0.95 + 0.000001x$ and $p_2(x) = 0.95 + 0.0000005x$.

6.3 Distortionary Tax

In the case of distortionary tax, equation 5.6 can be written as:

$$\rho VSL = \frac{1}{Hp'(G_I)} \tag{6.7}$$

where $\rho = 1 + \varepsilon_{l\tau^*}$ and $VSL = \frac{u(c^*_\tau(\tau), l^*_\tau(\tau))}{p(G_I)u'(c^*_\tau(\tau), l^*_\tau(\tau))}$. Given our analysis in section 5, ρ could be greater or lower than 1, which means that the direction of bias with using average VSL is uncertain under distortionary tax.

7 Policy Implications

The policy implications stem from the results of this paper focus on three different aspects: distributional weights, the marginal cost of public fund (MCPF) and the transfer of VSL.

7.1 Distributional weights

The use of average population VSL in the benefit measure of project evaluation has proven to be insensitive to individual heterogeneities and the imperfection of tax systems. Adler (2016) has already pointed out the insensitivity of cost-benefit analysis (CBA) to distributional concerns. Following the literature, one way to address this concern is to incorporate "distributional weights" into CBA.[13]

Currently the official guide for CBA in the UK (Treasury, 2003) recommend the use of marginal utility of consumption as distributional weights. However, this weighting scheme is over-simplified by not considering any non-consumption attributes. Adler (2016) states that different social welfare preferences of the decision maker should also be taken into account in choosing the proper weights. Our analysis adds to the literature by shedding light on how distributional weights should be adjusted in the presence of imperfect taxation.

The analyses in section 6 suggest that, distributional weights are subject to change under different heterogeneities (respectively wealth and risk) and different tax schemes (uniform and income tax). In terms of wealth heterogeneity, if taxation is perfect, no weights is needed. However, under imperfect taxation, subject to different tax schemes, the weights should correspond to the relative marginal utility of wealth $\lambda_i = \frac{u'(w_i - t^*_U)}{\sum_j u'(w_j - t^*_U)}$ (uniform tax) or $\mu_i = \frac{\sum_j w_j u'_i(w_i(1-\tau^*))}{\sum_i w_i u'_i(w_i(1-\tau^*))}$ (income tax). Moreover, the current guidelines for distributional weights do not consider individual heterogeneity in risks. Yet, when risk heterogeneity is coupled with uniform taxation, distributional weights should also be adjusted by the rate of risk sharing $\phi_i = \frac{p_i(G_U)}{\sum_j p_j(G_U)}$. Even when there is perfect taxation,

[13]For a full review of the literature, see Adler (2016).

if the risk reduction rate in the population is heterogeneous, a distributional weight corresponding to the relative marginal rate of risk reduction $\theta_i = \frac{p_i'(G_F)}{\sum_j p_j'(G_F)}$ should also be implemented.

7.2 Marginal Cost of Public Fund

It is well known that distortionary taxation creates additional cost to agents and welfare loss to the society, which is measured by MCPF. In CBA, MCPF is incorporated in the cost side of the evaluation (Brent, 2007). In practice, countries adopt different values of MCPF in their guidelines for CBA. For example, the U.S. Office of Management and Budget (OMB) recommends to use a MCPF of 1.25[14] and the European Union uses a default MCPF of 1.[15] However, our analysis in section 5 clearly shows the distortion stemmed from the tax system has an direct impact on the decision of optimal tax rate. Therefore, MCPF should not be limited to evaluating the cost when it also plays a role in optimizing the benefit. Moreover, proposition 4 also suggests that without further knowledge of the individual preference between consumption and labor effect, there is no systematic evidence that MCPF should always be greater than 1.

7.3 VSL Transfer

VSL is used in CBAs for various scenarios. However, it is inefficient and costly to separately calculate VSLs for each particular case. Thus, a common practice is to take the VSL estimate under a similar setting, and then quantitatively adjust the value to fit the particular case, known as "benefit transfer". Hammitt and Robinson (2011) look at the "benefit transfer" between high and low income populations, where VSL is often adjusted by the income elasticity of the populations in question. Our analysis suggests that, in addition to the income elasticity, inequality as well as the imperfection in the tax system also play important roles.

First of all, transferring VSL from one scenario to another may encounter the issue of increasing inequality. For instance, consider VSL transfer within the same population but to a different project, say from a clean water project to a transportation safety project.

[14] Circular No. A-94 Revised, artical 11. https://obamawhitehouse.archives.gov/omb/circulars_a094

[15] Guide to Cost-Benefit Analysis of Investment Projects. http://ec.europa.eu/regional_policy/sources/docgener/guides/cost/guide2008_en.pdf

Assume that for transportation related activities, one person may face significantly different baseline risk from another person, given her travel preference and patterns. Yet in the clean water project, all citizens have access to the same water source and are exposed to similar risks. Thus, given everything else equal, the two cases face different levels of risk inequality. We have seen from figure 4.1 that increasing risk inequality would change the optimal level of safety provision with perfect taxation. Thus, in such cases, the level of risk inequality should also be taken into account in the benefit transfers. Similarly, when transferring VSL from one population to another, income elasticity can only account for the inter-population wealth inequality, but not the intra-population wealth inequality. Proposition 2 and remark 3 have detailed assertions of this case.

Secondly, the imperfections in the tax systems also need to be considered in transferring VSL. The populations in question may not only be subject to different levels of wealth and risk inequality, but also have significantly different taxation systems. The results of this paper suggest that under first best, income and uniform taxation, the optimal levels of public safety differ. In the current environment of global decision making, especially in combating climate change, VSLs of each country are needed to accurately calculate the cost of carbon emissions. Therefore, knowing what the effective tax systems are in the target countries and adjust the VSLs corresponding to the imperfections in taxation is also important.

In sum, transferring VSL in various contexts needs careful consideration. Based on the theoretical analysis, we can not systematically adjust the value upwards or downwards without further investigation of the situation.

A Appendix

A.1 Bequest Motive

With a bequest motive, the expected utility of each individual is as follows:

$$p_i(G)u(w_i - t_i) + (1 - p_i(G))v(w_i - t_i) \tag{A.1}$$

where u and v are concave state dependent utilities of consumption in the states where the individual lives or dies respectively. As is common in the literature, we could consider the case that $v = ku$ for some $k \in [0, 1)$ (Kaplow, 2005; Viscusi & Evans, 1990). This means that the utility if you die is proportionally lower than the utility if you survive. Therefore, for each individual, we can write $\pi_i(G) = k + (1-k)p_i(G)$, $\pi(\cdot) > 0$, $\pi'(\cdot) > 0$, $\pi''(\cdot) \leq 0$. It is straight forward that all results of the paper carry out with this particular case.

A.2 Second Order Conditions

In order to have the second order condition satisfied for the maximization problem, we must have the Hessian of equation 4.4 to be negative definite. This would require that $M_{11} < 0$ ($\frac{\partial^2 L}{\partial t_1^2} < 0$) and $M_{22} > 0$ ($\frac{\partial^2 L}{\partial t_1^2}\frac{\partial^2 L}{\partial t_2^2} - (\frac{\partial^2 L}{\partial t_1 \partial t_2})^2 > 0$).

The first condition is easy to show. For the second condition, denote:

$$A_1 = p_1''u_1, A_2 = p_2''u_2, B_1 = p_1'u_1', B_2 = p_2'u_2', C_1 = p_1u_1'', C_2 = p_2u_2''$$

If

$$(A_1 + A_2)(C_1 + C_2) - (B_1 - B_2)^2 - 2B_1C_2 - 2B_2C_1 + C_1C_2 > 0$$

then the second order condition is satisfied globally. A sufficient condition would be $-C_2 < B_2 - B_1 < C_1$.

References

Adler, M. D. (2008). Risk equity: a new proposal. *Harvard Environmental Law Review*, *32*, 1.

Adler, M. D. (2016). Cost-benefit analysis and distributional weights: An overview. *Review of Environmental Economics & Policy*, *10*(2), 264–285.

Andersson, H., & Treich, N. (2011). The value of a statistical life. In A. De Palma, R. Lindsey, E. Quinet, & R. Vickerman (Eds.), *The Handbook of Transport Economics.* Edward Elgar Publishing.

Armantier, O., & Treich, N. (2004). Social willingness to pay, mortality risks and contingent valuation. *Journal of Risk and Uncertainty*, *29*(1), 7–19.

Atkinson, A. B., & Stern, N. H. (1974). Pigou, taxation and public goods. *The Review of Economic Studies*, *41*(1), 119–128.

Ballard, C., & Fullerton, D. (1992). Distortionary taxes and the provision of public goods. *Journal of Economic Perspectives*, *6*(3).

Bergstrom, T., Blume, L., & Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, *29*(1), 25–49.

Brent, R. J. (2007). Applied Cost-Benefit Analysis. Edward Elgar Publishing.

Browning, E. K. (1976). The marginal cost of public funds. *Journal of Political Economy*, *84*(2), 283–298.

Drèze, J. (1962). L'utilitè sociale d'une vie humaine. *Revue Française de Recherche Opèrationelle*, *6*(93), 93–118.

Eeckhoudt, L. R., & Hammitt, J. K. (2001). Background risks and the value of a statistical life. *Journal of Risk and Uncertainty*, *23*(3), 261–279.

Eeckhoudt, L. R., & Hammitt, J. K. (2004). Does risk aversion increase the value of mortality risk? *Journal of Environmental Economics and Management*, *47*(1), 13–29.

Foncel, J., & Treich, N. (2005). Fear of ruin. *Journal of Risk and Uncertainty*, *31*(3), 289–300.

Gaube, T. (2000). When do distortionary taxes reduce the optimal supply of public goods? *Journal of Public Economics*, *76*(2), 151–180.

Hammitt, J. K., & Robinson, L. A. (2011). The income elasticity of the value per statistical life: transferring estimates between high and low income populations.

Journal of Benefit-Cost Analysis, *2*(1), 1–29.

Jones-Lee, M. W. (1974). The value of changes in the probability of death or injury. *Journal of Political Economy*, *82*, 835–849.

Kaplow, L. (2005). The value of a statistical life and the coefficient of relative risk aversion. *Journal of Risk and Uncertainty*, *31*(1), 23–34.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, *58*(1), 53–73.

Manski, C. F. (2014). Identification of income–leisure preferences and evaluation of income tax policy. *Quantitative Economics*, *5*(1), 145–174.

Meghir, C., & Phillips, D. (2010). Labour supply and taxes. In *Dimensions of tax design: The mirrlees review* (pp. 202–274). Oxford University Press Oxford, UK, and New York.

Pigou, A. C. (1947). *A Study in Public Finance*. Macmillan and co., limited, 1947.

Pratt, J. W., & Zeckhauser, R. J. (1996). Willingness to pay and the distribution of risk and wealth. *Journal of Political Economy*, *104*(4), 747–763.

Samuelson, P. A. (1954). The pure theory of public expenditure. *The Review of Economics and Statistics*, *36*(4), 387–389.

Stiglitz, J. E., & Dasgupta, P. (1971). Differential taxation, public goods, and economic efficiency. *The Review of Economic Studies*, *38*(2), 151–174.

Treasury, H. M. (2003). *The Green Book: Appraisal and Evaluation in Central Government* (Tech. Rep.). Her Majesty's Treasury.

U.S.EPA. (2010). *The Guidelines for Preparing Economic Analyses* (Tech. Rep.). United States Environmental Protection Agency.

U.S.EPA. (2011). *The Benefits and Costs of the Clean Air Act from 1990 to 2020* (Tech. Rep.). U.S. Environmental Protection Agency Office of Air and Radiation.

U.S.EPA. (2016). *Valuing Mortality Risk Reductions for Policy: a Meta-analytic Approach* (Tech. Rep.). United States Environmental Protection Agency.

Viscusi, W. K., & Aldy, J. E. (2003). The value of a statistical life: a critical review of market estimates throughout the world. *Journal of Risk and Uncertainty*, *27*(1), 5–76.

Viscusi, W. K., & Evans, W. N. (1990). Utility functions that depend on health status: estimates and economic implications. *The American Economic Review*, 353–374.